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Extension of the flow-rate-of-strain tensor formulation of plasma rotation theory to non-axisymmetric tokamaks

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A systematic formalism for the calculation of rotation in non-axisymmetric tokamaks with 3D magnetic fields is described. The Braginskii $\Omega\tau$-ordered viscous stress tensor formalism, generalized to accommodate non-axisymmetric 3D magnetic fields in general toroidal flux surface geometry, and the resulting fluid moment equations provide a systematic formalism for the calculation of toroidal and poloidal rotation and radial ion flow in tokamaks in the presence of various non-axisymmetric "neoclassical toroidal viscosity" mechanisms. The relation among rotation velocities, radial ion particle flux, ion orbit loss, and radial electric field is discussed, and the possibility of controlling these quantities by producing externally controllable toroidal and/or poloidal currents in the edge plasma for this purpose is suggested for future investigation. © *2015 AIP Publishing LLC*.
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I. INTRODUCTION

Rotation in tokamak plasmas is an important topic of current research (e.g., the recent review of Ref. 1). Plasma rotation has been demonstrated to stabilize resistive wall modes (e.g., Ref. 2) and to correlate with energy confinement (e.g., Ref. 3), and rotation shear is widely believed to stabilize microinstabilities and thereby reduce diffusive energy transport (e.g., Refs. 4 and 5). Evidence of self-driven "intrinsic" rotation (e.g., Ref. 6) provides hope that these benefits of rotation can be extended to future large tokamaks without the necessity of large external torques.

The early "neoclassical" theories (e.g., Refs. 7–9) for rotation in tokamaks were developed on the basis of an assumed toroidal axisymmetry in a 2D magnetic field geometry. However, there have long been theoretical predictions of mechanisms (field ripple, error fields, magnetic instabilities, etc.) for the production of toroidal asymmetries in the magnetic field (e.g., Refs. 10–28). The damping of toroidal rotation by toroidal field ripple (e.g., Refs. 13 and 14) and the "magnetic braking" of toroidal rotation by a static n/m = 1/1 error field (Ref. 15) have been demonstrated experimentally. Such "classical" toroidal non-axisymmetric mechanisms for momentum damping have come to be collectively identified as "neoclassical toroidal viscosity" (NTV).

To the extent that the NTV mechanisms can be treated as triggers for producing viscosity, the resulting viscosities can be calculated from the fluid rate of strain tensor using neoclassical and/or NTV viscosity coefficients. In order to make such a fluid calculation of the effect of various NTV mechanisms on rotation in a tokamak, it is first necessary to generalize the fluid viscosity tensor representation to accommodate 3D magnetic fields and toroidal asymmetries.

We have previously adapted the Braginskii rotation theory based on the decomposition of the flow rate-of-strain tensor[29] from x-y-z geometry to a circular plasma toroidal flux surface geometry[30] and developed such an axisymmetric tokamak rotation theory[7,32] for circular cross section tokamaks, which we have subsequently generalized to an elongated flux surface geometry representation with Shafranov shift.[32,33] These "flow-rate-of-strain tensor" rotation theories have predicted measured rotation velocities relatively well,[31,33] except in the edge, but have not yet represented the NTV effects associated with the 3D toroidally asymmetric magnetic field geometry. The purpose of this paper is to utilize our recent generalization of the Braginskii stress tensor to a 3D, non-axisymmetric magnetic field geometry in order to construct a neoclassical plus NTV fluid rotation theory for tokamaks.

II. COORDINATE SYSTEM

We define a general right hand orthogonal (ψ, p, ϕ) flux surface coordinate system for an axisymmetric tokamak with differential length elements $(dl_\psi = h_\psi d\psi, dl_p = h_p dp, dl_\phi = h_\phi d\phi)$ and unit vectors $(\hat{n}_\psi = \nabla\psi/|\nabla\psi|, \hat{n}_\varphi = R\nabla\varphi, \hat{n}_p = \hat{n}_\varphi \times \hat{n}_\psi)$ where ψ is a radial-like flux surface variable associated with enclosed magnetic flux, p is a poloidal-like angular variable, and φ is the toroidal angle. The h_α are metric scale factors particular to the geometry (e.g., for a cylindrical system $(d\ell_r = dr, d\ell_\theta = rd\theta, dl_z = dz)$ and $(h_r = 1, h_\theta = r, h_z = 1)$). The 3D magnetic field structure and the non-axisymmetry will be represented in this axisymmetric coordinate system by allowing 3D radial (ψ) components of the magnetic field and a toroidal dependence of variables. In other words, we use a toroidally axisymmetric coordinate system in a fluid calculation, but allow 3D magnetic field and toroidal dependence of the variables. We note that this differs from the practice of some researchers who calculate NTV kinetically by representing the 3D magnetic "perturbation" as a perturbed particle Lagrangian.[16,21–24] Various specific axisymmetric and non-axisymmetric coordinate systems can be found in the literature.[11–28]

III. TOROIDAL ROTATION

In such a coordinate system, the toroidal angular momentum balance equation which governs toroidal rotation V_φ is

$$nmR\frac{\partial V_\varphi}{\partial t}+nmR[(\mathbf{V}\cdot\nabla)\mathbf{V}]_\varphi+R(\nabla p)_\varphi+R(\nabla\cdot\Pi)_\varphi \\ =enR(E_\varphi+V_\psi B_\theta-V_\theta B_\psi)+RF_\varphi+R\left(\mathbf{n}_\varphi\cdot\mathbf{S}^1-mS^0V_\varphi\right), \tag{1}$$

where R is the major radius from the tokamak centerline to a point in the plasma, E_φ and B_φ are toroidal electric and magnetic fields, F_φ is the toroidal component of the interspecies collisional friction force, $(\nabla\cdot\Pi)_\varphi$ is the toroidal component of the divergence of the viscosity tensor, $(\nabla p)_\varphi$ is the toroidal component of the pressure gradient, and S^1_φ and S^0 are toroidal momentum and particle sources or sinks. Note the new $V_\theta B_\psi$ term due to the 3D magnetic field. Note also that the p-subscript refers to the poloidal direction, but also that the quantity p in the equation is the pressure.

A. Viscosity

Braginskii's arguments[29] that three independent types of motion cause viscous forces that produce corrections to the Maxwellian distribution function used to calculate fluid theory viscosity are summarized in Ref. 35. These simple physical arguments indicate that gradients along the magnetic field lines of flows directed along the field lines produce large "parallel" momentum fluxes proportional to the parallel derivative of the parallel velocity that are independent of the magnetic field, with a coefficient that is proportional to the viscosity in the absence of the field, $\eta_0\simeq nm\lambda^2/\tau$, where λ is the distance traveled between 90° scattering events and τ is the ion-ion scattering time in a collisional plasma. In low collisionality plasmas and with particle trapping in magnetic wells these definitions are generalized. If the parallel velocity varies in a direction perpendicular to the field, there is a flux of parallel momentum in that perpendicular direction that is proportional to the perpendicular gradient of the parallel velocity in that perpendicular direction and to $\eta_\perp\simeq nm{r_L}^2/\tau=\eta_0/(\Omega\tau)^2$, where $\Omega=eB/m$ is the gyro-frequency and $r_L=V_{th}/\Omega$ is the gyroradius. An x-gradient in the y-velocity (where x and y are both perpendicular to the field) also produces a momentum flux of y-momentum is the x-direction but now with a mean collisional displacement of $r_L=V_{th}/\Omega$, so that the characteristic step size is the gyroradius rather than the collisional mean free path, resulting in a perpendicular viscosity coefficient is $\eta_\perp\approx\eta_0/(\Omega\tau)^2$. When the velocity in one perpendicular direction (say y) varies in the other perpendicular direction (x) or in the field direction (z) there are fluxes of perpendicular (y) momentum in the x- and z-directions proportional to these gradients and $\eta_\perp$. Finally, when the velocity in one perpendicular direction (y) varies in the other perpendicular direction (x) of in the parallel direction (z) there is an imbalance in the momentum flux caused by the gyromotion, which is proportional to the x-gradient of y-velocity gradient and to $\eta_\Omega\simeq 2nmV_{th}r_L\simeq\eta_0/\Omega\tau$. The original Braginskii derivation[29] in x-y-z geometry has been generalized to toroidal flux surface geometry,[31,32] to plasmas in the banana-plateau collision regime[7] and to elongated plasma geometry.[33,34]

Braginski showed that all three of the above types of flow gradients effects can be represented in the flow rate-of-strain tensor, which may be written for the generalized flux surface coordinate system of this paper[35]

$$W_{\alpha\beta}\equiv\hat{n}_\alpha\cdot\nabla\mathbf{V}\cdot\hat{n}_\beta+\hat{n}_\beta\cdot\nabla\mathbf{V}\cdot\hat{n}_\alpha-\frac{2}{3}\delta_{\alpha\beta}\nabla\cdot\mathbf{V} \\ =\left(\frac{\partial V_\beta}{\partial l_\alpha}+\sum_k\Gamma^\alpha_{\beta k}V_k\right)+\left(\frac{\partial V_\alpha}{\partial l_\beta}+\sum_k\Gamma^\alpha_{\beta k}V_k\right) \\ -\frac{2}{3}\delta_{\alpha\beta}\nabla\cdot\mathbf{V}, \tag{2}$$

where the Christoffel symbols are defined in terms of the metric elements of the coordinate system

$$\Gamma^\alpha_{\beta k}\equiv\frac{1}{h_\beta h_k}\left(\frac{\partial h_\beta}{\partial\xi_k}\delta_{\alpha\beta}-\frac{\partial h_k}{\partial\xi_\beta}\delta_{\alpha k}\right),(\xi_1=\psi,\xi_2=p,\xi_3=\phi). \tag{3}$$

The rate-of-strain tensor can, of course, be decomposed into terms corresponding to these three "parallel," "gyroviscous," and "perpendicular" components with associated viscosity coefficients, which differed by several orders of magnitude $(\eta_0\gg\eta_{3,4}\gg\eta_{1,2})$

$$\pi_{\alpha\beta}=-\eta_0W^0_{\alpha\beta}+[\eta_3W^3_{\alpha\beta}+\eta_4W^4_{\alpha\beta}]-[\eta_1W^1_{\alpha\beta}+\eta_2W^2_{\alpha\beta}] \\ \equiv\pi^0_{\alpha\beta}+\pi^{34}_{\alpha\beta}+\pi^{12}_{\alpha\beta}, \tag{4}$$

where denoting $f_\alpha\equiv B_\alpha/|B|$ and using the Einstein summation convention, the $W^n_{\alpha\beta}$ are

$$W^0_{\alpha\beta}\equiv\frac{3}{2}\left(f_\alpha f_\beta-\frac{1}{3}\delta_{\alpha\beta}\right)\left(f_\mu f_\nu-\frac{1}{3}\delta_{\mu\nu}\right)W_{\mu\nu}, \\ W^1_{\alpha\beta}\equiv\left(\delta^\perp_{\alpha\mu}\delta^\perp_{\beta\nu}+\frac{1}{2}\delta^\perp_{\alpha\beta}f_\mu f_\nu\right)W_{\mu\nu}, \\ W^2_{\alpha\beta}\equiv\left(\delta^\perp_{\alpha\mu}f_\beta f_\nu+\delta^\perp_{\beta\gamma}f_\alpha f_\mu\right)W_{\mu\nu}, \\ W^3_{\alpha\beta}=\frac{1}{2}\left(\delta^\perp_{\alpha\mu}\varepsilon_{\beta\gamma\nu}+\delta^\perp_{\beta\nu}\varepsilon_{\alpha\gamma\mu}\right)f_\gamma W_{\mu\nu}, \\ W^4_{\alpha\beta}=\frac{1}{2}(f_\alpha f_\mu\varepsilon_{\beta\gamma\nu}+\varepsilon_{\alpha\gamma\mu}f_\beta f_\nu)f_\gamma W_{\mu\nu}, \tag{5}$$

with $\varepsilon_{\alpha\beta\gamma}$ being the antisymmetric unit tensor, $\delta_{\alpha\beta}$ being the Kroneker delta function, and $\delta^\perp_{\alpha\beta}\equiv\delta_{\alpha\beta}-f_\alpha f_\beta$. Note that the $f_\alpha\equiv B_\alpha/|B|$ are defined as the position-dependent ratios of the $\alpha=\psi,p,\varphi$ components of the magnetic field to the to the total magnetic field strength at various locations in the axisymmetric coordinate system defined in Sec. II.

Braginskii[29] derived fluid viscosity coefficients for a collisional plasma $\eta_0\simeq n_iT_i\tau_i\gg\eta_{3,4}\simeq n_iT_i\tau_i/\Omega_i\tau_i\gg\eta_{1,2}\simeq n_iT_i\tau_i/(\Omega_i\tau_i)^2$, where τ_i is the inverse ion-ion collision frequency and $\Omega_i=e_iB/m_i$ is the ion gyrofrequency. Shaing[7] (and others) extended the parallel viscosity coefficient to the collisionless banana and plateau regimes

$$\eta_0 = \frac{n_i m_i V_{thi} q R \varepsilon^{-3/2} \nu_{ii}^*}{(1+\varepsilon^{-3/2}\nu_{ii}^*)(1+\nu_{ii}^*)} \equiv n_i m_i V_{thi} q R f_i(\nu_{ii}^*), \quad (6)$$

where $\nu_{ii}^* \equiv \nu_{ii} q R / V_{thi}$, $\nu_{ii} = \tau_i^{-1}$ is the ion-ion collision frequency and V_{thi} is the ion thermal speed. The gyroviscous coefficients $\eta_{3,4}$ are independent of collision frequency, and the perpendicular viscosity coefficients are too small to affect anything since $\Omega_i \tau_i \gg 1$ (but they also have been extended to low collisionality).

Working out the leading order parallel viscosity contributions for the components of the rate-of-strain tensors[36] yields an expression for the leading order parallel viscous stress tensor components

$$\pi^0_{\alpha,\beta} = -\frac{3}{2}\eta_0\left(f_\alpha f_\beta - \frac{1}{3}\delta_{\alpha\beta}\right)H^0, \quad (7)$$

where

$$H^0 \equiv \left[\begin{array}{l}
\left(f_\psi f_\psi - \frac{1}{3}\right)\left\{\frac{4}{3}\frac{\partial V_\psi}{\partial l_\psi} - \frac{2}{3}\left(\frac{\partial V_p}{\partial l_p} + \frac{\partial V_\phi}{\partial l_\phi}\right) + 2\left(\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial l_p}V_p + \frac{1}{h_\psi}\frac{\partial h_\psi}{\partial l_\phi}V_\phi\right)\right\} + \\
\left(f_p f_p - \frac{1}{3}\right)\left\{\frac{4}{3}\frac{\partial V_p}{\partial l_p} - \frac{2}{3}\left(\frac{\partial V_\psi}{\partial l_\psi} + \frac{\partial V_\phi}{\partial l_\phi}\right) + 2\left(\frac{1}{h_p}\frac{\partial h_p}{\partial l_\psi}V_\psi + \frac{1}{h_p}\frac{\partial h_p}{\partial l_\phi}V_\phi\right)\right\} + \\
\left(f_\phi f_\phi - \frac{1}{3}\right)\left\{\frac{4}{3}\frac{\partial V_\phi}{\partial l_\phi} - \frac{2}{3}\left(\frac{\partial V_p}{\partial l_p} + \frac{\partial V_\psi}{\partial l_\psi}\right) + 2\left(\frac{1}{h_\phi}\frac{\partial h_\phi}{\partial l_\psi}V_\psi + \frac{1}{h_\phi}\frac{\partial h_\phi}{\partial l_p}V_p\right)\right\} + \\
2f_\psi f_p\left\{\frac{\partial V_\psi}{\partial l_p} + \frac{\partial V_p}{\partial l_\psi} - \left(\frac{1}{h_p}\frac{\partial h_p}{\partial l_\psi}V_p + \frac{1}{h_\psi}\frac{\partial h_\psi}{\partial l_p}V_\psi\right)\right\} + \\
2f_p f_\phi\left\{\frac{\partial V_\phi}{\partial l_p} + \frac{\partial V_p}{\partial l_\phi} - \left(\frac{1}{h_p}\frac{\partial h_p}{\partial l_\phi}V_p + \frac{1}{h_\phi}\frac{\partial h_\phi}{\partial l_p}V_\phi\right)\right\} + \\
2f_\psi f_\phi\left\{\frac{\partial V_\phi}{\partial l_\psi} + \frac{\partial V_\psi}{\partial l_\phi} - \left(\frac{1}{h_\psi}\frac{\partial h_\psi}{\partial l_\phi}V_\psi + \frac{1}{h_\phi}\frac{\partial h_\phi}{\partial l_\psi}V_\phi\right)\right\}
\end{array}\right]. \quad (8)$$

The presence of radial magnetic field components is represented in these expressions by the $f_\psi(\psi,p,\varphi) \equiv B_\psi(\psi,p,\varphi)/|B(\psi,p,\varphi)|$ terms, and non-axisymmetry is further represented by the $\partial()/\partial l_\phi$ terms, where () is any such quantity so appearing in Eq. (8).

Following Ref. 36, the toroidal viscous force can be written

$$(\nabla\cdot\Pi)_\varphi = \left[\frac{1}{h_p h_\varphi}\frac{\partial(h_p h_\varphi \pi_{\psi\varphi})}{\partial \ell_\psi} - \frac{1}{h_\psi}\frac{\partial h_\psi}{\partial \ell_\varphi}\pi_{\psi\psi}\right] + \left[\frac{1}{h_\psi h_\varphi}\frac{\partial(h_\psi h_\varphi \pi_{p\varphi})}{\partial \ell_p} - \frac{1}{h_p}\frac{\partial h_p}{\partial \ell_\varphi}\pi_{pp}\right]$$
$$+ \left[\frac{1}{h_\psi h_p}\frac{\partial(h_p h_\psi \pi_{\varphi\varphi})}{\partial \ell_\varphi} + \frac{1}{h_\varphi}\frac{\partial h_\varphi}{\partial \ell_\psi}\pi_{\varphi\psi} + \frac{1}{h_\varphi}\frac{\partial h_\varphi}{\partial \ell_p}\pi_{\varphi p}\right]. \quad (9)$$

B. Viscous damping of toroidal angular momentum

In the flux surface average (FSA) of the toroidal angular momentum balance of Eq. (1), the viscous damping of toroidal angular momentum is represented by the flux surface average $\langle\rangle$ of the toroidal component of the viscous torque, which can be written

$$\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\rangle = \frac{1}{V'}\frac{\partial}{\partial\psi}\left(V'\langle R^2\nabla\phi\cdot\Pi\cdot\nabla\psi\rangle\right)$$
$$= \left\langle \frac{1}{Rh_p}\frac{\partial}{\partial l_\psi}\left(R^2 h_p \pi_{\psi\phi}\right) + B_p\frac{\partial}{\partial l_p}\left(\frac{R\pi_{p\phi}}{B_p}\right)\right\rangle, \quad (10)$$

where $\langle A\rangle \equiv \oint_\varphi d\ell_\varphi \oint_p d\ell_p \int_\psi^{\psi+d\psi} d\ell_\psi A(\psi,p,\varphi)/\oint_\varphi d\ell_\varphi \oint_p d\ell_p \int_\psi^{\psi+d\psi} d\ell_\psi$ and V' is the differential volume between flux surfaces. It can be shown[37] that the flux surface average of the second, $\pi^0_{p\phi}$ term in Eq. (10) is a perfect differential and must vanish from the requirement of single-valuedness, and hence there is no contribution of the leading order parallel viscosity to the toroidal angular momentum viscous damping from this term even if $\pi^0_{p\phi}$ is toroidally asymmetric. From Eq. (7), $\pi^0_{\psi\varphi} \equiv 0$ in the absence of a 3D (radial) component of the magnetic field (i.e., for $f_\psi \equiv B_\psi/|B| = 0$), so the first, $\pi^0_{\psi\varphi}$ term in Eq. (1) vanishes for $f_\psi \equiv B_\psi/|B| = 0$ but would survive for $f_\psi \equiv B_\psi/|B| \neq 0$. Only if there are 3D fields such that $f_\psi \equiv B_\psi/|B| \neq 0$, can there be a parallel viscosity contribution to toroidal angular momentum damping. If $f_\psi \equiv B_\psi/|B| = 0$ everywhere, the largest remaining contributions to toroidal angular momentum damping in axisymmetric tokamaks are due to the gyroviscous $\pi^{34}_{\psi\phi}$ and $\pi^{34}_{p\phi}$ terms[30–33] with viscosity coefficients $\eta_{34} \ll \eta_0$.

However, when $f_\psi \neq 0$, leading order (η_0) terms survive in Eq. (10), yielding a general form for the NTV viscous damping term

$$\langle R^2\nabla\varphi\cdot\nabla\cdot\Pi_0\rangle = -\frac{3}{2}\left\langle\frac{1}{Rh_p}\frac{\partial\left(R^2h_p\eta_0 f_\psi f_\varphi H^0\right)}{\partial\ell_\psi}\right\rangle = \frac{3}{2}\langle R\eta_0 f_\psi f_\varphi H^0 L_\psi^{-1}\rangle, \tag{11}$$

$$L_\psi^{-1} \equiv -\left\{\frac{1}{H^0}\frac{\partial H^0}{\partial\ell_\psi}+\frac{1}{f_\psi}\frac{\partial f_\psi}{\partial\ell_\psi}+\frac{1}{f_\varphi}\frac{\partial f_\varphi}{\partial\ell_\psi}+\frac{1}{\eta_0}\frac{\partial\eta_0}{\partial\ell_\psi}+\frac{1}{h_p}\frac{\partial h_p}{\partial\ell_\psi}+\frac{2}{R}\frac{\partial R}{\partial\ell_\psi}\right\}, \tag{12}$$

is a composite of inverse radial gradient scale-lengths of flow velocities, densities, temperature, magnetic fields, and the flux surface geometry. Note that the sign of $f_\varphi \equiv B_\varphi/|B|$ is >0 when the toroidal field is parallel to the plasma current and <0 when it is anti-parallel, in the representation of this paper. Various representations of $f_\psi \equiv B_\psi/|B|$ have been discussed in the literature (e.g., Refs. 11–28). These equations are reduced to a more familiar form for a "circular" plasma model in the Appendix.

The magnitude (η_0) and velocity dependence (H^0) of $\langle R^2\nabla\varphi\cdot\nabla\cdot\Pi_0\rangle$ of Eq. (8) are quite different from the magnitude and velocity dependence of the neoclassical toroidal angular momentum damping term $\langle R^2\nabla\varphi\cdot\nabla\cdot\Pi_{34}\rangle \simeq -\langle(1/rR)(\partial/\partial r)(R^3\eta_4\partial(V_\varphi/R)/\partial\theta)\rangle$ for an axisymmetric tokamak.[31] Because $\eta_0/\eta_{34} \simeq \Omega_i\tau_i \simeq 10^3 - 10^4$ the viscous damping due to 3D toroidal asymmetries $\langle R^2\nabla\varphi\cdot\nabla\cdot\Pi_0\rangle$ can be $\geq$ the gyroviscous damping due to poloidal asymmetries $\langle R^2\nabla\varphi\cdot\nabla\cdot\Pi_{34}\rangle$, even for small $f_\psi \equiv B_\psi/|B| \ll 1$.

We note that some other derivations of neoclassical rotation theory[8,9,17,19] have employed a gyroradius ($\delta = r_L/r < 1$) ordering instead of the Braginskii $\Omega\tau$ viscosity ordering $\eta_0 \gg \eta_{34} \gg \eta_{12}$, leading to neglect of the gyroviscous term which enters at a higher δ order, albeit with a much larger magnitude $(\eta_{34} \gg \eta_{12})$, than the perpendicular viscosity term that is retained. The Braginskii $\Omega\tau$ viscosity ordering is clearly more appropriate for rotation problems in that the larger gyroviscous term enters at a higher order than the smaller perpendicular viscosity term in the gyroradius ordering scheme. Note that the gyroviscous term arises in the gyroradius ordering in Refs. 8, 9, 17, and 19 but is neglected because it enters at a higher order in δ.

C. Toroidal inertial torque

The representation of the toroidal inertial torque in Eq. (1) in the general flux surface geometry of Sec. II is[65]

$$Rnm[(\mathbf{V}\cdot\nabla)\mathbf{V}]_\varphi = Rnm\left[\begin{array}{l}\left[V_\psi\dfrac{\partial V_\varphi}{\partial\ell_\psi}+V_\varphi V_\psi\left(\dfrac{1}{h_\varphi}\dfrac{\partial h_\varphi}{\partial\ell_\psi}-\dfrac{1}{h_\psi}\dfrac{\partial h_\psi}{\partial\ell_\varphi}\right)\right]\\ +\left[V_p\dfrac{\partial V_\varphi}{\partial\ell_p}+V_\varphi V_p\left(\dfrac{1}{h_\varphi}\dfrac{\partial h_\varphi}{\partial\ell_p}-\dfrac{1}{h_p}\dfrac{\partial h_p}{\partial\ell_\varphi}\right)\right]+V_\varphi\dfrac{\partial V_\varphi}{\partial\ell_\varphi}\end{array}\right]. \tag{13}$$

IV. POLOIDAL ROTATION

Poloidal rotation is governed by the poloidal angular momentum balance equation

$$\begin{aligned}&rnm\frac{\partial V_p}{\partial t}+rnm[(\mathbf{V}\cdot\nabla)\mathbf{V}]_p+r(\nabla p)_p+r(\nabla\cdot\Pi)_p\\&\quad= rne(E_p-V_\psi B_\varphi+V_\varphi B_\psi)+rF_p+r\left(\hat{\mathbf{n}}_p\cdot\mathbf{S}^1-mV_pS^0\right),\end{aligned} \tag{14}$$

where $r(\psi,p,\varphi)$ is the minor radius from the minor axis at toroidal angle φ out to the point (ψ,p,φ) on the flux surface ψ. Note that allowing for a 3D magnetic field introduces an additional $B_\psi V_\varphi$ into Eq. (14).

The leading order poloidal viscous force is[36]

$$[\nabla\cdot\Pi]_p^0 = -\frac{3}{2}\left[\begin{array}{l}\left\{\dfrac{\partial\left(\eta_0 f_\psi f_p H^0\right)}{\partial\ell_\psi}+\dfrac{\partial\left(\eta_0\left(f_p^2-\frac{1}{3}\right)H^0\right)}{\partial\ell_p}+\dfrac{\partial\left(\eta_0 f_\varphi f_p H^0\right)}{\partial\ell_\varphi}\right\}+\\ \eta_0 H^0\left\{\begin{array}{l}f_\psi f_p\left(\dfrac{1}{h_p}\dfrac{\partial h_p}{\partial\ell_\psi}+\dfrac{1}{h_\varphi}\dfrac{\partial h_\varphi}{\partial\ell_\psi}\right)-\left(f_\psi^2-\dfrac{1}{3}\right)\left(\dfrac{1}{h_\psi}\dfrac{\partial h_\psi}{\partial\ell_p}\right)+\\ \left(f_p^2-\dfrac{1}{3}\right)\left(\dfrac{1}{h_\psi}\dfrac{\partial h_\psi}{\partial\ell_p}+\dfrac{1}{h_\varphi}\dfrac{\partial h_\varphi}{\partial\ell_p}\right)+f_p f_\varphi\left(\dfrac{1}{h_p}\dfrac{\partial h_p}{\partial\ell_\varphi}\right)+f_p f_\psi\left(\dfrac{1}{h_p}\dfrac{\partial h_p}{\partial\ell_\psi}\right)+\\ f_\varphi f_p\left(\dfrac{1}{h_p}\dfrac{\partial h_p}{\partial\ell_\varphi}+\dfrac{1}{h_\psi}\dfrac{\partial h_\psi}{\partial\ell_\varphi}\right)-\left(f_\varphi^2-\dfrac{1}{3}\right)\left(\dfrac{1}{h_\varphi}\dfrac{\partial h_\varphi}{\partial\ell_p}\right)\end{array}\right\}\end{array}\right]. \tag{15}$$

Comparison of Eqs. (7)–(9) with Eq. (15) reveals that the toroidal and poloidal viscous forces caused by non-axisymmetric configurations will have similar magnitudes and dependences on the poloidal and toroidal flows, a consequence of the parallel viscosity component being the leading order contribution in both.

The poloidal inertial force can be written[36]

$$nm[(\mathbf{V}\cdot\nabla)\mathbf{V}]_p = nm\begin{bmatrix}\left[V_\psi\dfrac{\partial V_p}{\partial \ell_\psi} + V_pV_\psi\left(\dfrac{1}{h_p}\dfrac{\partial h_p}{\partial \ell_\psi} - \dfrac{1}{h_\psi}\dfrac{\partial h_\psi}{\partial \ell_p}\right)\right] + \\ \left[V_p\dfrac{\partial V_p}{\partial \ell_p}\right] + \left[V_\varphi\dfrac{\partial V_p}{\partial \ell_\varphi} + V_pV_\varphi\left(\dfrac{1}{h_p}\dfrac{\partial h_p}{\partial \ell_\varphi} - \dfrac{1}{h_\varphi}\dfrac{\partial h_\varphi}{\partial \ell_p}\right)\right]\end{bmatrix}. \tag{16}$$

V. RADIAL ION PARTICLE FLUX

The "radial" (ψ) component of the momentum balance is

$$nm\frac{\partial V_\psi}{\partial t} + nm[(\mathbf{V}\cdot\nabla)\mathbf{V}]_\psi + (\nabla p)_\psi + (\nabla\cdot\Pi)_\psi = ne(E_\psi - V_\varphi B_p + V_p B_\varphi) + F_\psi + \left(\hat{\mathbf{n}}_\psi\cdot\mathbf{S}^1 - mV_\psi S^0\right). \tag{17}$$

Formally, this radial component of the momentum balance equation governs the evolution of the radial particle velocity in response to the included forces. In practice, this equation is commonly used to determine the radial electric field after assuming the radial flows to be small $(V_\psi \ll V_p < V_\varphi)$ and ignoring the source terms and the radial inertial and viscous forces as small. It is not clear that this is always appropriate in the edge plasma, and it will be of interest to systematically determine the radial velocity from the radial momentum balance and determine the radial electric field from Ohm's law.

The radial viscous force can be written[36]

$$\begin{aligned}(\nabla\cdot\Pi)_\psi = &\left[\frac{1}{h_ph_\varphi}\frac{\partial(h_ph_\varphi\pi_{\psi\psi})}{\partial\ell_\psi} + \frac{1}{h_\psi}\frac{\partial h_\psi}{\partial\ell_p}\pi_{\psi p} + \frac{1}{h_\psi}\frac{\partial h_\psi}{\partial\ell_\varphi}\pi_{\psi\varphi}\right] \\ &+\left[\frac{1}{h_\psi h_\varphi}\frac{\partial(h_ph_\psi\pi_{p\psi})}{\partial\ell_p} - \frac{1}{h_p}\frac{\partial h_p}{\partial\ell_\psi}\pi_{pp}\right] \\ &+\left[\frac{1}{h_\psi h_p}\frac{\partial(h_ph_\psi\pi_{\varphi\psi})}{\partial\ell_\varphi} - \frac{1}{h_\varphi}\frac{\partial h_\varphi}{\partial\ell_\psi}\pi_{\varphi\varphi}\right],\end{aligned} \tag{18}$$

and the radial inertial force can be written[36]

$$nm[(\mathbf{V}\cdot\nabla)\mathbf{V}]_\psi = nm\begin{bmatrix}\left[V_\psi\dfrac{\partial V_\psi}{\partial \ell_\psi}\right] + \left[V_p\dfrac{\partial V_\psi}{\partial \ell_p} + V_\psi V_p\left(\dfrac{1}{h_\psi}\dfrac{\partial h_\psi}{\partial \ell_p} - \dfrac{1}{h_p}\dfrac{\partial h_p}{\partial \ell_\psi}\right)\right] \\ + \left[V_\varphi\dfrac{\partial V_\psi}{\partial \ell_\varphi} + V_\psi V_\varphi\left(\dfrac{1}{h_\psi}\dfrac{\partial h_\psi}{\partial \ell_\varphi} - \dfrac{1}{h_\varphi}\dfrac{\partial h_\varphi}{\partial \ell_\psi}\right)\right]\end{bmatrix}. \tag{19}$$

VI. ION ORBIT LOSS

The fraction of "thermalized" ions that are able to access loss orbits which remove them from the plasma increases as the ions are transported radially outward. The minimum energy $E_{0\min}$ at which these ions are "ion-orbit-lost" can be calculated at each radius from conservation of canonical angular momentum, magnetic moment, and energy. The increasing with radius loss fractions of outward transport fluxes of ions and energy can then be calculated[38–40]

$$F_{orb}(\rho) \equiv \frac{N_{loss}}{N_{tot}} = \frac{\int_{-1}^{1}\left[\int_{V_{0\min(\zeta_0)}}^{\infty} V_0^2 f(V_0)dV_0\right]d\zeta_0}{2\int_0^\infty V_0^2 f(V_0)dV_0} = \frac{\int_{-1}^{1}\Gamma\left(\dfrac{3}{2},\varepsilon_{\min}(\rho,\zeta_0)\right)d\zeta_0}{2\Gamma\left(\dfrac{3}{2}\right)}, \tag{20}$$

and

$$E_{orb}(\rho) \equiv \frac{E_{loss}}{E_{total}} = \frac{\int_{-1}^{1}\left[\int_{V_{0min}(\zeta_0)}^{\infty}\left(\dfrac{1}{2}mV_0^2\right)V_0^2 f(V_0)dV_0\right]d\zeta_0}{\int_{-1}^{1}\left[\int_0^\infty\left(\frac{1}{2}mV_0^2\right)V_0^2 f(V_0)dV_0\right]d\zeta_0} = \frac{\int_{-1}^{1}\Gamma\left(\dfrac{5}{2},\varepsilon_{\min}(\rho,\zeta_0)\right)d\zeta_0}{2\Gamma\left(\dfrac{5}{2}\right)}, \tag{21}$$

where $V_{0\min}(\zeta_0) = \sqrt{2E_{0\min}(\zeta_0)/m}$ is the minimum speed for which particles with velocity direction cosine ζ_0 (relative to the B-field) can be lost, and an initially Maxwellian distribution has been assumed. The quantity $\varepsilon_{\min} = E_{0\min}/kT_{ion}$, and Γ in Eqs. (20) and (21) is the gamma function. A compensating return current of ions inwards from the scrape-off-layer is required in order to maintain charge neutrality. This compensating current replaces the lost energetic particle with a thermalized particle with small momentum. Thus, ion orbit loss directly affects the momentum and energy balances but not the particle balance.

In the "predictive" formalism presented in this paper, the ion-orbit particle losses can be represented by a momentum loss term $S_\psi^1 = -2(\partial F_{orb}/\partial \ell_\psi) n V_\psi$ in the radial momentum balance Eq. (17), where the "2" accounts for the ion orbit loss of an outward flowing ion plus the inward compensating return current. Alternatively, Eq. (17) can be solved for the radial particle flux $\Gamma_\psi = nV_\psi$ without taking ion orbit loss into account and then corrected for the latter $\hat{\Gamma}_\psi = \Gamma_\psi(1 - 2F_{orb})$. In the interpretation of experimental density profiles (e.g., Refs. 38 and 40) in which Γ_ψ is determined by integrating the continuity equation, the radial particle flux being transported in the plasma is determined by similarly correcting for ion orbit loss $\hat{\Gamma}_\psi = \Gamma_\psi(1 - 2F_{orb})$. In the interpretive analyses (e.g., Refs. 39 and 41) the total energy fluxes are also corrected for ion orbit loss $\hat{Q}_\psi = Q_\psi(1 - E_{orb})$, and this would be appropriate with the formalism of this paper.

Counter-current directed ions are lost predominantly, resulting in a co-current intrinsic rotation of the ions remaining in the plasma,[42–46] which is given by

$$\Delta V_\varphi(\rho) = \int_{-1}^{1} d\zeta_0 \left[\int_{V_{\min}(\zeta_0)}^{\infty} (V_0 \zeta_0) V_0^2 f(V_0) dV_0 \right]_\rho = \frac{\int_{-1}^{1} \zeta_0 \Gamma(2, \varepsilon_{\min}(\rho, \zeta_0)) d\zeta_0}{\Gamma(2)\pi^{\frac{3}{2}}} \sqrt{\frac{2kT_{ion}(\rho)}{m}}, \tag{22}$$

that contributes to the edge plasma rotation.

Ion orbit losses, f_{nbi}^{iol}, and intrinsic velocities for fast neutral beam injected ions can be calculated from similar considerations.

VII. ION DENSITY AND ENERGY BALANCE EQUATIONS

Solution of Eqs. (1), (14), and (17) requires also a determination of the density from the continuity equation

$$\frac{\partial n}{\partial t} + \frac{1}{h_p h_\varphi} \frac{\partial (h_p h_\varphi n V_\psi)}{\partial \ell_\psi} + \frac{1}{h_\psi h_\varphi} \frac{\partial (h_\psi h_\varphi n V_p)}{\partial \ell_p} + \frac{1}{h_p h_\psi} \frac{\partial (h_p h_\psi n V_\varphi)}{\partial \ell_\psi} = \mathrm{N}_{nbi} + n_e \nu_{ion} = S^0, \tag{23}$$

where N_{nbi} is the local neutral beam particle source rate and ν_{ion} is the ionization frequency of incoming neutral atoms, and a solution of the ion and electron energy balance equations

$$\begin{aligned} \nabla \cdot \mathbf{Q}_i \simeq \frac{\partial Q_{\psi i}}{\partial \ell_\psi} &= -\frac{\partial}{\partial t}\left(\frac{3}{2} n_i T_i\right) + q_{nbi}^i (1 - f_{nbi}^{iol}) \\ &\quad - q_{ie} - n_i n_o^c \langle \sigma v \rangle_{cx} \frac{3}{2} (T_i - T_o^c), \\ \nabla \cdot \mathbf{Q}_e \simeq \frac{\partial Q_{\psi e}}{\partial \ell_\psi} &= -\frac{\partial}{\partial t}\left(\frac{3}{2} n_e T_e\right) + q_{nbi}^e (1 - f_{nbi}^{iol}) \\ &\quad + q_{ie} - n_e n_z L_z(T_e), \end{aligned} \tag{24}$$

where $Q = (3/2)TnV + q$ and the conductive energy flux is approximated by the Fourier heat relation $\mathbf{q} = -n\chi\nabla T$. The second terms on the right are the neutral beam or other heating terms (reduced to account for ion orbit loss), the q_{ie} is the collisional ion-electron energy exchange rate, and the last terms represent charge-exchange cooling of ions and radiation cooling of electrons.

VIII. RADIAL ELECTRIC FIELD

Solving the ion and electron momentum balance equations using an interspecies friction term of the form $\mathbf{F}_{ei} = -nm\nu_{ei}(\mathbf{V}_e - \mathbf{V}_i) = -\eta n^2 e^2 (\mathbf{V}_e - \mathbf{V}_i) = ne\eta \mathbf{j}$ leads to[47]

$$\mathbf{E} = \eta \mathbf{j} - \mathbf{u} \times \mathbf{B} - \frac{\nabla p_e}{ne} + \frac{\mathbf{j} \times \mathbf{B}}{ne}, \tag{25}$$

where $\mathbf{u} = (n_i m_i \mathbf{V}_i + n_e m_e \mathbf{V}_e)/(n_i m_{ii} + n_e m_e)$, with a sum over ion species "i" being understood. Making use of the leading order plasma force balance $\mathbf{j} \times \mathbf{B} = \nabla p = \nabla p_i + \nabla p_e$ yields another form of Ohm's Law for the determination of the electric field in a rotating plasma

$$\mathbf{E} = \eta \mathbf{j} - \mathbf{u} \times \mathbf{B} + \frac{\nabla p_i}{ne}, \tag{26}$$

where E is the electric field, η is the plasma resistivity, $\mathbf{j}$ is the plasma current density and $\mathbf{u}$ is the mass rotation velocity of the plasma. Equation (26) is a convenient form for the determination of the radial electric field that is produced by the rotation velocities and the ion pressure gradient discussed above.

However, Eq. (25) provides an insight into how the radial electric field (and hence the quantities such as ion orbit loss, radial ion flux, and rotation that depend upon it) could be controlled by creating a poloidal or toroidal current in the plasma, most likely in the edge plasma. Further pursuit of this obviously interesting and potentially important topic is beyond the scope of this paper, but is definitely recommended for future investigation.

IX. SUMMARY AND DISCUSSION

The axisymmetric tokamak rotation theory based on the Braginskii $\Omega\tau$ ordering of viscous forces and the flow rate-of-strain tensor has been extended to non-axisymmetric tokamak plasmas. The most important finding is that the leading order parallel viscosity contribution to the FSA of the toroidal momentum damping rate does not vanish when the possibility of radial magnetic fields is retained in the development. This paper provides a powerful formalism for investigating rotation applicable to the study of non-axisymmetric tokamaks.

It remains now to identify the specific f_ψ representations, viscosity coefficients and ordering schemes for specific non-axisymmetries and to reduce the above formalism to practical computational models by eliminating terms that are unimportant to the specific application. We give one example in the Appendix. The scope of such specific models ranges from (i) semi-analytical expressions for toroidal angular momentum damping rates through (ii) approximate solutions of the toroidal and poloidal momentum balance equations that can be used to interpret momentum transport in experiments to (iii) coupled 3D numerical calculations of the toroidal and poloidal rotation velocities, the radial particle flux and the radial electric field. Such further development can build upon the significant existing literature on NTV (e.g., Refs. 16–28) to represent the radial magnetic fields produced by field ripple, instabilities, field error, etc., and to model various mechanisms for producing parallel viscosity (i.e., for modeling η_0).

One useful extension of the present basic formalism of this paper would be inclusion of the heat flux contribution to the flow rate-of-strain tensor.[48] Such an extension would make the viscosity tensor more relevant for the "low-rotation" plasmas expected in future tokamaks.

The possibility of controlling the radial electric field by imposing externally controllable poloidal or toroidal currents in the plasma that is suggested by Eq. (25) is potentially very important and is recommended for future investigation.

APPENDIX: CIRCULAR PLASMA MODEL PROBLEM

In order to gain better insight, Eqs. (7), (8), and (11) can be reduced into a practical NTV formulae for tokamak plasmas with "circular" cross sections $R = R_0 + r\cos\theta = R_0(1+\varepsilon\cos\theta)$ and with $V_\psi \ll V_p < V_\phi$ and $f_\phi \approx 1$ assumed. The coordinate system reduces to the more familiar form

$$d\ell_\psi \approx h_r\partial r = \partial r,\quad d\ell_p = h_p\partial p \approx h_\theta\partial\theta = r\partial\theta,\quad d\ell_\phi = h_\phi\partial\phi = R\partial\phi. \tag{A1}$$

Leading to a reduced form of Eq. (8)

$$H^0 \equiv -\frac{2}{3}\left(f_rf_r-\frac{1}{3}\right)\left(\frac{1}{r}\frac{\partial V_\theta}{\partial\theta}+\frac{\partial V_\phi}{R\partial\phi}\right)+\left(f_\theta f_\theta-\frac{1}{3}\right)\left[\frac{4}{3}\frac{1}{r}\frac{\partial V_\theta}{\partial\theta}-\frac{2}{3}\left(\frac{\partial V_\phi}{R\partial\phi}\right)\right]+\frac{2}{3}\left[\frac{4}{3}\frac{\partial V_\phi}{R\partial\phi}-\frac{2}{3}\frac{1}{r}\left(\frac{\partial V_\theta}{\partial\theta}\right)+2\left(\frac{1}{Rr}\frac{\partial R}{\partial\theta}V_\theta\right)\right]$$
$$+2f_rf_\theta\left[\frac{\partial V_\theta}{\partial r}-\left(\frac{1}{r}V_\theta\right)\right]+2f_\theta\left[\frac{1}{r}\frac{\partial V_\phi}{\partial\theta}+\frac{\partial V_\theta}{R\partial\phi}-\frac{1}{Rr}\frac{\partial R}{\partial\theta}V_\phi\right]+2f_r\left[\frac{\partial V_\phi}{\partial r}-\left(\frac{1}{R}\frac{\partial R}{\partial r}V_\phi\right)\right]. \tag{A2}$$

We further neglect any poloidal dependence of velocities for simplicity, so that $V_\phi(r,\theta,\phi)\approx\bar V_\phi(r,\phi)$ and $V_\theta(r,\theta)\approx\bar V_\theta(r)$, to reduce Eqs. (7) and (8) to

$$\pi^0_{r\phi}\approx-\frac{3}{2}\eta_0f_rH^0=-\frac{3}{2}\eta_0f_r\left[\begin{array}{l}-\frac{2}{3}\left(f_rf_r-\frac{1}{3}\right)\frac{1}{R}\frac{\partial\bar V_\phi(r,\phi)}{\partial\phi}-\frac{2}{3}\left(f_\theta f_\theta-\frac{1}{3}\right)\frac{1}{R}\frac{\partial\bar V_\phi(r,\phi)}{\partial\phi}\\+\frac{8}{9}\frac{1}{R}\frac{\partial\bar V_\phi(r,\phi)}{\partial\phi}+\frac{4}{3}\frac{1}{Rr}\frac{\partial R}{\partial\theta}\bar V_\theta(r)+2f_rf_\theta\frac{\partial\bar V_\theta(r)}{\partial r}-2f_rf_\theta\frac{1}{r}\bar V_\theta(r)\\-2f_\theta\frac{1}{Rr}\frac{\partial R}{\partial\theta}\bar V_\phi(r,\phi)+2f_r\frac{\partial\bar V_\phi(r,\phi)}{\partial r}-2f_r\frac{1}{R}\frac{\partial R}{\partial r}\bar V_\phi(r,\phi)\end{array}\right]. \tag{A3}$$

We further make practical assumptions $f_r\equiv B_r/B\neq f_r(r)$, $f_\theta\equiv B_\theta/B\neq f_\theta(r)$, $B=B(r)\neq B(r,\theta)$, $f_r\le O(10^{-3})\sim O(\varepsilon^3)$, $f_\theta\le O(10^{-1})\sim O(\varepsilon^1)$, and $\partial R_0/\partial r\le O(10^{-1})\sim O(\varepsilon^1)$, $O(f_r\partial\bar V_\phi(r,\phi)/\partial r)\sim O(\partial\bar V_\phi(r,\phi)/\partial\phi)$, and neglect 2nd derivatives and the Shafranov shift, to reduce Eq. (11) to

$$\langle R^2\nabla\phi\cdot\nabla\cdot\pi^0\rangle\approx\frac{3}{r}f_r\left[\begin{array}{l}\bar V_\phi(r,\phi)f_r\varepsilon\left(\eta_0+\frac{r}{2}\frac{\partial\eta_0}{\partial r}\right)+\bar V_\theta f_rf_\theta\left(2\eta_0\frac{\varepsilon}{2}+\frac{\partial\eta_0}{\partial r}R_0\right)\\+\frac{\partial\bar V_\phi(r,\phi)}{\partial\phi}\frac{2}{27}\left(\eta_0+r\frac{\partial\eta_0}{\partial r}\right)-\frac{\partial\bar V_\phi(r,\phi)}{\partial r}f_rR_0\left(\eta_0+r\frac{\partial\eta_0}{\partial r}\right)\end{array}\right]. \tag{A4}$$

The third and fourth terms on the right-hand side can be converted into toroidal and radial gradient length scale lengths, similar to Eq. (12), to combine them with the first term to represent the entire toroidal velocity contribution in the form of Eqs. (11) and (12). Equation (A4) contains contributions from both toroidal and poloidal velocities. Practical modeling issues must be worked out for the proper representation of the toroidal and radial derivatives of the toroidal velocity, the third and fourth terms. We plan to test a few practical models in the near future.

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