

A neoclassical model for toroidal rotation and the radial electric field in the edge pedestal

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A model for the calculation of toroidal rotation velocities and the radial electric field in the edge pedestal of tokamaks is described. The model is based on particle and momentum balance and the use of the neoclassical gyroviscous expression for the toroidal viscous force. Predicted toroidal rotation velocities in the edge pedestal are found to agree with measured values to within about a factor of 2 or less, for a range of DIII-D [Luxon, Nucl. Fusion **42**, 614 (2002)] edge pedestal conditions. © 2004 American Institute of Physics. [DOI: 10.1063/1.1710520]

I. INTRODUCTION

The coincidence of changes in the local plasma rotation and radial electric field in the plasma edge, the *L-H* (low-to-high mode) transition, and the establishment of the *H*-mode edge pedestal is well established experimentally (see, e.g., Refs. 1–4), leading to the widely held opinion that the plasma rotation and the radial electric field are important phenomena affecting the *L-H* transition and the formation of the *H*-mode pedestal. The leading paradigm for how these phenomena act to effect the *L-H* transition is via the creation of a local region of strong $E \times B$ shear that stabilizes turbulent transport.⁵

This situation motivates us to develop a first-principles calculation model for the rotation velocities and radial electric field in the edge of tokamak plasmas in order to understand the dependence of these quantities on the edge plasma and operating parameters. We have previously presented a model for the calculation of poloidal velocities and poloidal density asymmetries in the edge plasma,⁶ and the purpose of this paper is to present a complementary model for the calculation of toroidal velocities and the radial electric field.

The model presented in this paper and in Ref. 6 is based on fluid particle and momentum balance. Neoclassical physics is included through the use of neoclassical forms for the parallel viscous force (in Ref. 6), for the toroidal viscous force (this paper) and for the collisional momentum exchange among species (both papers). The development follows the same general lines as for our similar calculation model for rotation in the core plasma,⁷ but also takes into account atomic physics ionization sources and atomic physics momentum damping unique to the plasma edge, as well as the radial particle flux into the edge from the core (which is the dominant term driving rotation in the edge) and the gyroviscous momentum damping.

II. THEORY

The particle continuity equation for ion species *j* is

$$\nabla \cdot n_j \mathbf{v}_j = S_j, \quad (1)$$

where $S_j(r, \theta) = n_e(r, \theta)n_{j0}(r, \theta)\langle \sigma v \rangle_{\text{ion}} \equiv n_e(r, \theta)v_{\text{ion}}(r, \theta)$ is the ionization source rate of ion species *j* and n_{j0} is the local concentration of neutrals of species *j*. Taking the flux surface average of this equation yields $\langle (\nabla \cdot n_j \mathbf{v}_j)_r \rangle = \langle S_j \rangle$ because $\langle (\nabla \cdot n_j \mathbf{v}_j)_\theta \rangle = 0$ identically and $\langle (\nabla \cdot n_j \mathbf{v}_j)_\phi \rangle = 0$ by axisymmetry, which allows Eq. (1) to be written

$$(\nabla \cdot n_j \mathbf{v}_j)_\theta = S_j - \langle S_j \rangle \equiv \tilde{S}_j \quad (2)$$

when we make the assumption $(\nabla \cdot n_j \mathbf{v}_j)_r = \langle (\nabla \cdot n_j \mathbf{v}_j)_r \rangle + O(\epsilon)$.

Integration of this equation, in toroidal (r, θ, ϕ) coordinates, yields

$$\begin{aligned} n_j v_{\theta j} &= \frac{K_j \bar{B}_\theta + r \bar{B}_\theta \int_0^\theta (1 + \epsilon \cos \theta) \tilde{S}_j d\theta}{1 + \epsilon \cos \theta} \\ &\equiv [K_j(r) + I_j(r, \theta)] B_\theta(r, \theta), \end{aligned} \quad (3)$$

where $B_\theta = B_{\theta 0} / (1 + \epsilon \cos \theta)$, which ignores Shafranov shift effects, has been used and where $K_j = \langle n_j v_{\theta j} \rangle / \bar{B}_\theta \approx \bar{n}_j \bar{v}_{\theta j} / \bar{B}_\theta$ and the overbar denotes the average value over the flux surface. We note that using toroidal geometry and assuming poloidally uniform radial particle fluxes ignores some potentially important geometric effects in diverted tokamaks.

Subtracting $m_j v_j$ times Eq. (1) from the momentum balance equation for ion species *j* and noting that $(\nabla \cdot n_j \mathbf{v}_j)_r \ll (\nabla \cdot n_j \mathbf{v}_j)_\theta$ leads to

$$\begin{aligned} n_j m_j (\mathbf{v}_j \cdot \nabla) \mathbf{v}_j + \nabla p_j + \nabla \cdot \mathbf{p}_j \\ = n_j e_j (\mathbf{v}_j \times \mathbf{B}) + n_j e_j \mathbf{E} + \mathbf{F}_j + \mathbf{M}_j - n_j m_j \nu_{\text{at}}^j \mathbf{v}_j - m_j \tilde{S}_j \mathbf{v}_j, \end{aligned} \quad (4)$$

where \mathbf{F}_j represents the interspecies collisional friction, \mathbf{M}_j represents the external momentum input rate, and the last two terms represent the momentum loss rate due to elastic scattering and charge exchange with neutrals of all ion species k [$\nu_{\text{at}}^j = \sum_k n_{k0}^c (\langle \sigma v \rangle_{\text{el}} + \langle \sigma v \rangle_{\text{cx}})_{jk}$] and due to the introduction of ions with no net momentum via ionization of a neutral of species *j*. Only the “cold” neutrals that have not already suffered an elastic scattering or charge-exchange collision in the pedestal are included in ν_{at}^j . Equation (4) can

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be understood by noting that the conservative form of the inertia term $[\text{div}(nmv) = mv \cdot \text{div}(nv) + nm(v \cdot \text{del})v]$ appears in the usual momentum equation. When $mv \cdot \text{Eq. (1)}$ is subtracted from that momentum equation, the $mv \cdot \text{div}(nv)$ terms cancel and the ionization source term shows up.

Using the Lorentz form for the interspecies collisional friction

$$\mathbf{F}_j = -n_j m_j \sum_{k \neq j} \nu_{jk} (\mathbf{v}_j - \mathbf{v}_k) \quad (5)$$

and taking the flux surface average of the toroidal component of Eq. (4) yields a coupled set of equations for the toroidal velocities of the different ion species present plus the electrons

$$\left(\nu_{dj}^* + \sum_{k \neq j} \nu_{jk} \right) \nu_{\phi j} - \sum_{k \neq j} \nu_{jk} \nu_{\phi k} = \frac{(n_j e_j E_\phi^A + e_j \Gamma_j B_\theta + M_{\phi j})}{n_j m_j} \equiv y_j \quad (6)$$

where the total “drag” frequency ν_{dj}^* is given by

$$\bar{\nu}_{dj}^* \equiv \bar{\nu}_{dj} + \bar{\nu}_{atj} + \bar{\nu}_{ionj} \xi_j \quad (7)$$

which consists of a cross-field viscous momentum transport frequency formally given by

$$\bar{\nu}_{dj} \equiv \langle R^2 \nabla \phi \cdot \nabla \cdot \pi_j \rangle / \bar{R} \bar{n}_j m_j \bar{\nu}_{\phi j} \quad (8)$$

and of the two atomic physics momentum loss terms discussed previously, with the neutral ionization source asymmetry characterized by

$$\xi_j \equiv \langle R^2 \nabla \phi \cdot m_j \bar{S}_j \nu_{\phi j} \rangle / \bar{R} m_j \bar{S}_j \bar{\nu}_{\phi j}. \quad (9)$$

Since the condition $(n_{\text{carbon}} Z_{\text{carbon}}^2 / n_e) \gg (m_e / m_D)^{1/2} \approx 0.016$ is satisfied in most plasmas, the ion-electron collisions can be neglected relative to the ion-impurity collisions in Eq. (6). In the limiting case of a two-species ion-impurity (*i-I*) plasma, the two Eqs. (6) can be solved to obtain the toroidal rotation velocity of each species,

$$\nu_{\phi j} = \frac{[1 + (\nu_{dk}^* / \nu_{kj})] y_j + y_k}{[\{1 + (\nu_{dk}^* / \nu_{kj})\} \{1 + (\nu_{dj}^* / \nu_{jk})\} - 1]}. \quad (10)$$

The toroidal rotation is driven by the input beam torque ($RM_{\phi j}$), the input torque associated with the induced field ($Rn_j e_j E_\phi$), and by the internal torque due to the radial ion flow ($e_j B_\theta \Gamma_j$) which enter the y_j , and depends on the radial transfer rate of toroidal angular momentum (ν_{dj}^*) due to viscous, atomic physics and convective effects and on the interspecies momentum exchange rate (ν_{jk}).

The difference in toroidal rotation velocities of the two species is

$$\nu_{\phi j} - \nu_{\phi k} = \frac{(\nu_{dk}^* / \nu_{kj}) y_j - (\nu_{dj}^* / \nu_{jk}) y_k}{[\{1 + (\nu_{dk}^* / \nu_{kj})\} \{1 + (\nu_{dj}^* / \nu_{jk})\} - 1]}. \quad (11)$$

In order to actually evaluate the above equations it is necessary to specify the toroidal viscous force, $\langle R^2 \nabla \phi \cdot \nabla \cdot \pi \rangle$, which determines the viscous momentum transport frequency ν_{dj} , given by Eq. (8). There are three neoclassical

viscosity components—parallel, perpendicular, and gyroviscous. The “parallel” component of the neoclassical viscosity vanishes identically in the viscous force term, and the “perpendicular” component is several orders of magnitude smaller than the “gyroviscous” component⁸

$$\langle R^2 \nabla \phi \cdot \nabla \cdot \pi_j \rangle = \frac{1}{2} \tilde{\theta}_j G_j \frac{n_j m_j T_j}{e_j B_\phi} \frac{\nu_{\phi j}}{R} \equiv R n_j m_j \nu_{dj} \nu_{\phi j}, \quad (12)$$

where

$$\tilde{\theta}_j \equiv (4 + \tilde{n}_j^c) \tilde{\nu}_{\phi j}^s + \tilde{n}_j^s (1 - \tilde{\nu}_{\phi j}^c) \quad (13)$$

represents poloidal asymmetries and

$$G_j \equiv \frac{r}{\eta_{4j} \nu_{\phi j}} \frac{\partial(\eta_{4j} \nu_{\phi j})}{\partial r} \quad (14)$$

with the gyroviscosity coefficient $\eta_{4j} \approx n_j m_j T_j / e_j B$.

In order to evaluate Eq. (13) it is first necessary to calculate the sine and cosine components of the density and toroidal velocity poloidal variations over the flux surface. A low-order Fourier expansion of the densities and rotation velocities over the flux surface can be made, and Eq. (3) and the radial component of Eq. (4) can be used to relate the Fourier components of the rotation velocities for species j to the Fourier components of the density for that species. These results then can be used in the poloidal component of Eq. (4), the flux surface average of which with 1, $\sin \theta$ and $\cos \theta$ weighting then yields a coupled set of three nonlinear equations per species that can be solved numerically for the flux surface average poloidal velocities and the sine and cosine components of the ion density variations over the flux surface.⁶

We note that it has been suggested⁹ that the above expression for the gyroviscous toroidal force underestimates the momentum transport rate in regions of steep pressure gradients and low toroidal rotation (e.g., the edge pedestal) because of failure to take into account a drift kinetic correction not present in the original Braginskii derivation. Braginskii’s momentum equations are valid if the fluid velocities in the directions perpendicular and parallel to \mathbf{B} are much larger than the diamagnetic velocity and the diamagnetic velocity multiplied by B_ϕ / B_θ , respectively. Ordering arguments suggest that this is not the case in the absence of a large “external” source of momentum. It is not *a priori* clear if the Braginskii gyroviscous formulation is correct for the conditions of the plasma edge or needs to be supplemented by a heat flux term.¹⁰ In any case, the above equations have done well in predicting toroidal rotation (hence radial momentum transport) in the DIII-D core plasma,¹¹ which motivates us to investigate their predictions in the edge pedestal.

The flux surface average of the radial component of the momentum balance Eq. (4) yields

$$\bar{\nu}_{\phi j} = f_p^{-1} \bar{\nu}_{\theta j} - (\bar{P}'_j + \bar{\Phi}'_j), \quad (15)$$

where

$$f_p \equiv B_\theta / B_\phi, \quad \bar{P}'_j \equiv \frac{1}{\bar{n}_j e_j B_\theta} \frac{\partial \bar{p}_j}{\partial r},$$

TABLE I. DIII-D edge pedestal parameters ($R=1.74\text{--}1.78\text{ m}$, $a=0.60\text{--}0.62\text{ m}$, $B=1.5\text{--}2.1\text{ T}$, $I=1.0\text{--}1.6\text{ MA}$, $\kappa=1.7\text{--}2.0$, $\delta=0.13\text{--}0.86$) (Ref. 13).

	P_{nb} (MW)	n_e^{ped} ($10^{19}/\text{m}^3$)	T_e^{ped} (eV)	Δ_n (cm)	Δ_{T_e} (cm)	L_n (cm)	L_{T_e} (cm)	L_{T_i} (cm)	f_{carbon} (%)
A	5.1	4.0	1150	5.1	5.5	2.8	2.2	4.7	4.1
B	7.5	2.8	685	8.1	10.2	4.3	4.5	8.5	5.5
C	6.5	6.3	525	3.5	5.0	3.3	2.6	6.2	1.1
D	5.0	4.6	460	4.6	4.6	2.7	2.1	5.3	1.8
E	5.0	4.6	395	4.4	5.9	2.4	2.0	10.3	2.0
F	5.0	4.9	215	3.6	7.2	6.0	4.2	10.3	1.8
G	2.1	8.3	120	2.2	2.2	1.5	1.5	10.1	0.8

$$\bar{\Phi}' \equiv \frac{1}{B_\theta} \frac{\partial \phi}{\partial r} = -\frac{\bar{E}_r}{B_\theta}. \quad (16)$$

When Eq. (15) is used to eliminate $v_{\varphi j}$ from Eq. (6), the resulting equations can be summed over ion species (and the toroidal electron momentum equation can be used) to obtain an explicit expression for the radial electric field,

$$\frac{E_r}{B_\theta} = \frac{\sum_j^{\text{ions}} \{M_{\phi j} + n_j m_j \nu_{dj}^* (P_j' - f_p^{-1} v_{\theta j})\}}{\sum_j^{\text{ions}} n_j m_j \nu_{dj}^*}. \quad (17)$$

The local electric field depends on the total local input momentum deposition ($M_\varphi = \sum_j M_{\varphi j}$), the local radial pressure gradients (P_j'), the local poloidal velocities ($v_{\theta j}$), and the local values of the radial momentum transfer rates (ν_{dj}^*) due to viscous, atomic physics and convective effects.

III. COMPARISON WITH EXPERIMENT

We have used the above theory and the theory for poloidal rotation presented in Ref. 6 to calculate the rotation velocities and the electric field in the edge pedestals of the DIII-D H -mode plasmas described in Table I. (“ped” indicates value at the top of edge pedestal, Δ is the pedestal width, and L is the gradient scale length in the pedestal.)

We have evaluated Eqs. (10) and (17) for the toroidal velocities of the deuterium main ion and a carbon impurity ion species and for the radial electric field, respectively. We have also solved numerically for the poloidal rotation velocities and the sine and cosine components of the main and impurity ion species using Eqs. (22)–(24) of Ref. 6, derived from the poloidal momentum balance as described in the previous section.

The terms entering these equations were evaluated as follows. The viscous momentum transfer frequency ν_{di} was calculated from the neoclassical gyroviscous Eq. (12), with the poloidal asymmetry factors of Eq. (13) evaluated from poloidal momentum balance and with the factor G of Eq. (14) evaluated using experimental values of the radial gradient scale lengths in the edge pedestal. The radial particle flux was determined from particle balance, and the neutral beam momentum input in the pedestal was calculated directly. The friction terms involving the difference in ion and electron toroidal velocities were assumed to be negligible. The E_φ^A term and the pressure gradient terms were evaluated from experimental data.

TABLE II. Comparison of calculated and experimental rotation velocities (km/s) and radial electric fields (kV/m).

	$V_{\varphi c}^{\text{ex}}/V_{\text{th}}$	$V_{\varphi c}^{\text{ex}}$	$V_{\varphi c}^{\text{cal}}$	$V_{\varphi D}^{\text{cal}}$	$V_{\theta c}^{\text{ex}}$	$V_{\theta c}^{\text{cal}}$	$V_{\theta D}^{\text{cal}}$	E_r^{ex}	E_r^{cal}
A	0.1	6	8	6	-1	-0	-5	-42	-57
B	0.7	55	34	32	9	-0	-9	-15	-32
C	0.3	17	23	21	3	-2	-1	-13	-14
D	0.2	13	25	23	-2	-1	-2	-2	-15
E	0.3	17	25	23	-0	-1	-3	-7	-21
F	0.2	9	22	21	-1	-2	-1	-13	-6
G	0.4	13	30	28	3	4	-2	-2	-4

The neutral concentrations needed to evaluate ν_{at} and ν_{ion} and the recycling neutral influx needed to calculate the main ion Γ_D were obtained using a two-dimensional (2D) neutral transport calculation of fueling and recycling neutrals coupled to a “two-point” scrape-off layer and divertor plasma model and to a core plasma particle and power balance model.¹² The plasma ion flux to the divertor plate was recycled as neutral atoms (at a fraction of the incident ion energy) or molecules which were assumed to immediately dissociate into Franck–Condon atoms (at $\sim 2\text{ eV}$). These atoms were transported out of the divertor across the separatrix and into the plasma edge to produce a poloidally distributed neutral density which was averaged to evaluate ν_{at} and ν_{ion} . Measured plasma densities in the scrape-off layer and pedestal region were used in calculating the penetration of recycling neutrals. Atoms that were ionized inside the separatrix contributed to the neutral source used to calculate Γ_D , and atoms that were charge-exchanged or scattered were assumed to take on the energy of the ions at that location. Although the neutral transport calculation was well founded, the recycling neutral source was uncertain in these calculations. We normalized the calculations to experiment by adjusting the recycling source so that the calculated core fueling by neutral influx plus neutral beam resulted in a prediction of the line-average density that agreed with the experimental value. This model has been found to predict neutral densities that are in reasonable agreement with measured values in DIII-D and with Monte Carlo predictions.¹⁴

Determination of the carbon impurity Γ_C was more uncertain. The argument that in steady state the carbon outflux must equal the carbon influx and that the latter must be proportional to the deuterium outflux ($\Gamma_C = R\Gamma_D$) was used to evaluate Γ_C . The factor R involves the sputtering yield (in the range $0.01 < Y < 0.02$), the enhancement of the ion flux due to charge-exchange recycling neutrals and the reduction of the carbon flow to the plasma due to retention in the divertor, the calculation of which is beyond the scope of this paper. We used $R=0.01$, and checked that a factor of 2 difference in the value of R produced only about a 5% change in the calculated toroidal velocities.

The calculated and measured rotation velocities and radial electric fields are compared in Table II. Only the carbon rotation velocity is measured, and its separation into toroidal

and poloidal components introduces an uncertainty of a few km/s. The “experimental” radial electric field is actually calculated from the radial force balance Eq. (15) using the measured carbon velocity and pressure gradient. The calculated values are based on averaged parameters in the sharp gradient edge pedestal region, and the experimental values correspond to locations about midway in this pedestal region.

The most relevant comparison is probably between the measured and calculated values of the carbon toroidal rotation velocities, because of the large experimental error in the measured poloidal velocities which propagates into the calculation of the experimental radial electric field. The calculated and measured toroidal rotation velocities agree to within roughly a factor of 2 or better. Since the gyroviscous momentum transfer frequency ν_d was the dominant component of the total momentum transfer frequency ν_d^* , this agreement between measured and calculated toroidal rotation velocities indicates that neoclassical momentum transport theory is in reasonable agreement with experiment in the DIII-D edge pedestal, over a wide range of edge pedestal conditions. We note that the difference in calculated deuterium and carbon toroidal rotation velocities was on the order of 10%, so that the commonly made assumption that they are identical is reasonable.

For the deuterium main ion species the dominant term in the driving term y_D was the radial particle flux term $eB_\theta\Gamma_D$. Thus the “internal” torque due to the radial ion flux is the principal driver of toroidal rotation in the edge pedestal in these shots.

We also note that the measured carbon toroidal rotation speed was a significant fraction of the carbon thermal speed in the edge pedestal (i.e., $v_\phi \approx v_{th}$ is a more appropriate ordering than $v_\phi \ll v_{th}$).

IV. DISCUSSION

A model for ion toroidal velocities and the radial electric field in the edge pedestal region of tokamaks was presented. The model is based on particle and momentum balance and incorporates the neoclassical gyroviscous toroidal viscous force. The toroidal rotation is driven by the input beam torque ($RM_{\phi j}$), the input torque associated with the induced

field ($Rn_j e_j E_\phi$), and by the internal torque due to the radial ion flow ($e_j B_\theta \Gamma_j$), and depends on the radial transfer rate of toroidal angular momentum (ν_{dj}^*) due to viscous, atomic physics and convective effects and on the interspecies momentum exchange rate (ν_{jk}). The local electric field depends on the total local input momentum deposition ($M_\phi = \sum_j M_{\phi j}$), the local radial pressure gradients (P_j^θ), the local poloidal velocities ($v_{\theta j}$), and the local values of the radial momentum transfer rates (ν_{dj}^*) due to viscous, atomic physics and convective effects.

The calculation model that was introduced in this paper predicts carbon toroidal rotation velocities in the DIII-D edge pedestal to within about a factor of 2 or less, for a wide range of edge pedestal parameters. This result is consistent with the recent observation¹³ that the measured momentum transport frequency through the edge pedestal was within about a factor of 2 of the neoclassical gyroviscous prediction, over this same set of edge pedestal conditions. These results provide a measure of confidence in the calculation model for toroidal rotation in the edge pedestal that was presented in this paper.

Finally, we note other recent treatments of toroidal rotation in ALCATOR C-Mod Ohmic H modes from the viewpoint of neoclassical theory¹⁵ and accretion theory.¹⁶

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