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An Edge Pedestal Model

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Abstract

An edge pedestal model is proposed in which the density and temperature gradient scale lengths and the density and temperature widths in the edge pedestal are determined by the physics of: 1) the local transport and atomic physics processes and the particle and heat fluxes flowing through the pedestal; 2) MHD pressure gradient constraints; 3) neutral particle penetration into the plasma edge pedestal; and 4) MHD pedestal β-limit constraints.

1 Introduction

Although the physics of the edge pedestal is an area of intensive experimental [e.g. Ref. 1-7] investigation and theoretical research [e.g. Ref. 8-15], no model has yet emerged which provides a self-consistent, first-principles description of density and temperature gradient scale lengths (L_n, L_{Te}, L_{Ti}) and the corresponding widths (Δ_n,Δ_{Te}, Δ_{Ti}) of the sharp-gradient regions. The purpose of this paper is to propose such an edge pedestal model based on plasma and neutral particle transport constraints within the edge plasma, heat and particle fluxes flowing through the edge pedestal, and MHD constraints on the maximum allowable values of both the pressure gradient and the pressure in the edge pedestal.

2 Pedestal Transport and Atomic Physics Constraints

Average gradient scale lengths in the edge pedestal are determined from transport and atomic physics considerations [16]. An average density gradient scale length, L_n, can be defined by writing the average particle flux in the pedestal as $\Gamma_{\perp}^{a\mathrm{V}} = n\left(DL_n^{-1} + \mathrm{V}_p\right)$. $\Gamma_{\perp}^{a\mathrm{V}}$ can be related to the particle flux crossing the separatrix, $\Gamma_{\perp}^{sep}$, by integrating the particle balance equation over the pedestal region of width Δ_n in which a sharp density gradient exists and taking ionization of neutrals into account. The resulting expression for the density gradient scale length in the pedestal is

$$L_n = \frac{D}{(\Gamma_{\perp}^{sep}/n) - \mathrm{V}_p - \frac{1}{2}\Delta_n \nu_{ion}}. \tag{1}$$

Following a similar procedure for the ion and electron heat fluxes, taking into account convection, impurity radiation, ionization, charge-exchange and elastic scattering leads to

$$L_{Ti} = \frac{\chi_i}{\left[\left(\frac{Q_{\perp i}^{sep}}{nT_i} - \frac{5}{2}{}_c\frac{\Gamma_{\perp}^{sep}}{n}\right) + \frac{1}{2}\left(\Delta_{Ti}\frac{3}{2}\nu_{at}^{c} + \Delta_n\frac{5}{2}\nu_{ion}\right)\right]} \tag{2}$$

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and

$$L_{Te} \;=\; \frac{\chi_e}{\left[\left(\frac{Q_{\perp e}^{sep}}{nT_e} \;-\; \frac{5}{2}{}_c\frac{\Gamma_{\perp}^{sep}}{n}\right)+\frac{1}{2}\left(\Delta_{Te}\left(\frac{n_zL_z}{T_e}+\nu_{ion}\frac{E_{ion}}{T_e}\right)+\frac{5}{2}\Delta_n\nu_{ion}\right)\right]} \tag{3}$$

where $Q_{\perp}^{sep}$is the total heat flux crossing the separatrix, $\nu_{at}^{c}$is the charge-exchange plus elastic scattering frequency for "cold" neutrals which have not yet had a collision in the pedestal, and n_z and L_z are the density and radiation function for the impurities. The densities and temperatures in the above equations are understood to be the average values over the respective pedestal widths, which we calculate from $-(dn/dr)/n = L_n^{-1}$, etc.

$$\begin{aligned} n^{TB} \;&=\; n^{sep}\left(\tfrac{L_n}{\Delta_n}\right)\left(e^{\Delta_n/L_n}-1\right) \\ T_{i,e}^{TB} &= T_{i,e}^{sep}\left(\tfrac{L_{Ti,e}}{\Delta_{Ti,e}}\right)\left(e^{\Delta_{Ti,e}/L_{Ti,e}}-1\right) \end{aligned} \tag{4}$$

where 'sep' indicates the value on the separatrix or LCFS. These transport constraints have been applied, together with an empirical fit for the pedestal width, to a model problem and found to predict magnitudes and some trends of gradient scale lengths observed experimentally [16].

3 MHD Pressure Gradient Constraints

Nominal first stability ballooning mode theory places a limit on the maximum pressure gradient

$$\left(-\frac{dp}{dr}\right)_{\max} = \frac{B^2/2\mu_o}{q_{95}^2\;R}\;C_sS_o \tag{5}$$

where S_o is the shear calculated without taking into account any bootstrap current effects in the plasma edge and C_s is a constant of order unity. The presence of steep pressure gradients in the plasma edge will drive a bootstrap current, which will reduce the shear, which in turn will alter the maximum stable edge pressure gradient

$$\left(-\frac{dp}{dr}\right)_{\max} = \frac{B^2/2\mu_o}{q_{95}^2\;R}\;C_sA_s\left(s\right)\left[S_o-2\frac{j_{bs}}{\langle j\rangle}\right] \tag{6}$$

where $\langle j\rangle = I/\pi a^2\kappa$ is the average current density over the entire plasma cross section and j_{bs} is the bootstrap current density in the edge. The quantity $A_s\left(s\right)$is intended to contain the physics of the $s-\alpha$diagram for ballooning modes and peeling modes (e.g. [7,12]). The general effect of this physics is to reduce$A_s$below unity because of FLR stabilization of high-n ballooning modes [7], stabilization of all ballooning modes by ion diamagnetic drift effects [13], access to the second stable regime [7,12], etc. A major next task in the development of the edge pedestal model is the parameterization of $A_s\left(s\right)$. Two bootstrap current models are considered. A simple model (e.g. Miyamoto [17]), corrected for collisional effects by the factor [7] $f\left(\nu_i\right)=\left(1+\sqrt{\nu_i^2}\right)^{-1}$ is

$$j_{bs} = -\frac{\sqrt{\varepsilon}}{B_\theta}\,\frac{dp}{dr}\;\left(1+\sqrt{\nu_i^*}\right)^{-1} \tag{7}$$

and a more comprehensive model (Wesson [18]) is

$$j_{bs} = -\frac{\sqrt{2\varepsilon}\ p_e}{B_\theta\ D}\ \left[C_1 L_{pe}^{-1} + C_2 L_{pi}^{-1} + C_3 L_{Te}^{-1} + C_4 L_{Ti}^{-1}\right] \tag{8}$$

where the $C_n\left(\nu_i^*, \nu_e^*\right)$ contain collisionality effects and D is a function of ε [18]. The requirement for stability against ballooning (and peeling) modes

$$\left(-\frac{dp}{dr}\right) \leq \left(-\frac{dp}{dr}\right)_{\max} \tag{9}$$

can be written as a MHD constraint on the allowable values of the gradient scale lengths

$$L_p^{-1} \ \equiv \ \left(L_n^{-1} + \frac{T_e}{T_e + T_i}\ L_{Te}^{-1} + \frac{T_i}{T_e + T_i}\ L_{Ti}^{-1}\right) \leq \ L_{MHD}^{-1} \equiv \frac{1}{p}\left(-\frac{dp}{dr}\right)_{\max} \tag{10}$$

where

$$L_{MHD} \ = \ \frac{\left(\frac{p}{B_\theta^2/2\mu_o}\right)}{\frac{1}{2}\left(\frac{4\pi}{5\times 10^6 \mu_o}\right)^2} \frac{\varepsilon g^2}{1+\kappa^2} \frac{a}{C_s S_o}\ \text{(nominal)} \tag{11}$$

$$L_{MHD} \ = \ \frac{\left(\frac{p}{B_\theta^2/2\mu_o}\right)}{\frac{1}{2}\left(\frac{4\pi}{5\times 10^6 \mu_o}\right)^2} \frac{\varepsilon g^2}{1+\kappa^2} \frac{a(1+k_1)}{C_s A_s S_o}\ \text{(Miyamoto)} \tag{12}$$

$$L_{MHD} \ = \ \frac{\left(\frac{p}{B_\theta^2/2\mu_o}\right)}{\frac{1}{2}\left(\frac{4\pi}{5\times 10^6 \mu_o}\right)^2} \frac{\varepsilon g^2}{1+\kappa^2} \frac{a(1+k_2)}{C_s A_s\left[S_o - \frac{1}{2}\sqrt{2\varepsilon}\frac{a\kappa T_e}{D(T_e+T_i)}\left(\frac{p}{B_\theta^2/2\mu_o}\right)\left\{C_3 L_{Te}^{-1} + C_4 L_{Ti}^{-1}\right\}\right]}\ \text{(Wesson)} \tag{13}$$

corresponding to the use of Eq. (5), Eqs. (6) and (7), or Eqs. (6) and (8), respectively, to evaluate the maximum edge pressure gradient. Here

$$g \ \equiv \ \left[1 + \kappa^2\left(1 + 2\ \delta^2 - 1.2\ \delta^3\right)\right]\ \frac{[1.17 - 0.65\varepsilon]}{\left[1-\varepsilon^2\right]^2} \tag{14}$$

is a geometric factor in the definition of the safety factor $q_{95} \ = \ (5/2)\left(a^2/R\right)\left(\frac{B}{I_{MA}}\right) g$, and

$$k_1 \ \equiv \ \left[\frac{1}{2}\left(\frac{4\pi}{5\times 10^6 \mu_o}\right)^2 \frac{\kappa\left(1+\kappa^2\right)}{\sqrt{\varepsilon}\ g^2}\ C_s A_s\right]\ \frac{1}{1+\sqrt{\nu_i^*}} \tag{15}$$

and

$$k_2 \equiv \left[\frac{1}{2}\left(\frac{4\pi}{5\times 10^6 \mu_o}\right)^2 \frac{\kappa\,(1+\kappa^2)}{\sqrt{\varepsilon}\; g^2}\; C_s A_s\right] \frac{\sqrt{2}\; C_1\,(\nu_i^*)}{D} \tag{16}$$

are factors arising from the dependence of the bootstrap currents of Eqs. (7) and (8), respectively, on the pressure gradient.

4 Neutral Penetration Constraints on the Density Pedestal Width

The buildup in plasma density from the separatrix inward to the top of the density pedestal is due, at least in part, to the ionization of recycling and fueling neutrals inward across the separatrix, which implies that the limit of penetration of these neutrals into the plasma will play an important role in determining the density pedestal width, Δ_n. Using a simple diffusion theory model for inward neutral atom transport [19] and replacing the varying plasma density in the pedestal with an average value, n_{TB}, it can be shown [20] that the neutral atom density attenuates exponentially into the plasma with an effective mean free path $\lambda_0 = [n_{TB}(3\sigma_{tr}\sigma_{ion})^{1/2}]^{-1}$, where σ_{ion} is the ionization cross section, $\sigma_{tr} = \sigma_{ion} + \sigma_{cx}(1\text{-}\mu_{cx}) + \sigma_{el}(1\text{-}\mu_{el})$ is the 'transport' cross section, $\sigma_{cx}$and σ_{el} are the charge-exchange and elastic scattering cross sections, and μ_{cx} and μ_{el} are the average cosine of the neutral atom 'scattering' angle in charge-exchange and elastic scattering collisions with a plasma ion. Since the direction of the neutral atom emerging from a charge-exchange collision with a plasma ion is equally likely to be in any direction, $\mu_{cx} = 0$. For the elastic scattering of a neutral atom from an ion, $\mu_{el} \approx 2/3$A, where A = ion mass/neutral mass. A plausible approximation for the density width of the pedestal is

$$\Delta_n \simeq \lambda_o = \left[n^{TB}\sqrt{3\sigma_{tr}\sigma_{ion}}\right]^{-1} \tag{17}$$

5 MHD Pedestal β-Limit Constraint

A number of phenomena have been suggested (see Ref.[1]) as the cause for the relatively narrow width of the sharp pressure gradient region in the edge of H-mode plasmas. We suggest that a MHD stability β-limit on the maxium pedestal pressure determines the pedestal pressure width, Δ_p. For example, stability of the edge pedestal against MHD ideal pressure-driven surface modes imposes a maximum allowable value on the pedestal pressure [13]

$$p_{crit} = \frac{8}{3}\frac{B^2/2\mu_o}{q_{95}^2}\left(\frac{T_i}{2\,(T_i+T_e)}\right)^{1/3}\left(\frac{\rho_i}{R}\right)^{2/3}. \tag{18}$$

For a given edge pressure gradient scale length, L_p, and a given pressure at the separatrix (p_{sep} determined by divertor physics), the β-limit on the maximum allowable pedestal pressure, p_{crit}, results in the constraint

$$p_{crit} \leq n^{ped}\left(T_i^{ped} + T_e^{ped}\right) = n_{sep}e^{\Delta_p/L_n}\left(T_i^{sep}e^{\Delta_p/L_{Ti}} + T_e^{sep}e^{\Delta_p/L_{Te}}\right) \tag{19}$$

when p_{crit} is understood to pertain to the pressure at the top of pedestal, and in the constraint

$$p_{crit} \leq n^{TB}\left(T_i^{TB} + T_e^{TB}\right) \tag{20}$$

when p_{crit} is understood to pertain to the average pressure over the pedestal region. There is some ambiguity in how to relate the pressure pedestal width, Δ_p, to the component widths (Δ_n,Δ_{Te}, Δ_{Ti}). The dependence of L_p on $\Delta = (\Delta_n, \Delta_{Ti}, \Delta_{Te})$ is shown in Eqs. (1) - (3) and (9), and is due to alteration of particle and heat fluxes in the pedestal by atomic physics. We note that the critical pressure constraint of this section and the critical pressure gradient constraint of section 3 are inequality constraints. While it is likely that the pedestal may be limited by one of these constraints, so that the inequality constraint may be replaced by an equality, it is not clear that both of these constraints would be limiting simultaneously, in which case an additional constraint would be needed.

6 An Approximate Solution for the Pedestal Width

We assume $\Delta_n = \Delta_{Ti} = \Delta_{Te} = \Delta_{TB}$, set p_{TB}= (p_{ped}+ p_{sep})/2 = p_{crit} and solve L_p^{-1}=-(dp/dr)/p to obtain

$$\Delta_{TB} = L_p(\Delta_{TB})\,\ell n\left(2\frac{p_{crit}}{p_{sep}} - 1\right) \equiv L_p(\Delta_{TB})\,G\,. \tag{21}$$

Using Eqs.(1)-(3), (10) and (18) in Eq. (21) results in a quadratic equation in Δ_{TB}, which has the solution

$$\Delta_{TB} = \frac{b}{2a}\left[\sqrt{1 + \frac{4aG}{b^2}} - 1\right] \simeq \frac{G}{b}\left[1 - \left(\frac{aG}{b^2}\right) + \left(\frac{aG}{b^2}\right)^2 - \ldots\right]. \tag{22}$$

The second form of this result is valid when atomic physics effects are not dominant; i.e. when $\left|\frac{aG}{b^2}\right| < 1$. The constants in Eq. (22) are

$$a \equiv \frac{1}{2}\left[-\frac{\nu_{ion}}{D} + \gamma_i\frac{\left(\frac{3}{2}\nu_{at}^c + \frac{5}{2}\nu_{ion}\right)}{\chi_i} + \gamma_e\frac{\left(\frac{n_zL_z}{T_e} + \nu_{ion}\left\{\frac{E_{ion}}{T_e} + \frac{5}{2}\right\}\right)}{\chi_e}\right] \tag{23}$$

$$b \equiv \left[\frac{(\Gamma_\perp^{sep}/n - V_p)}{D} + \gamma_i\frac{\left(\frac{Q_{\perp i}^{sep}}{nT_i} - \frac{5}{2}\frac{\Gamma_\perp^{sep}}{n}\right)}{\chi_i} + \gamma_e\left(\frac{\left(\frac{Q_{\perp e}^{sep}}{nT_e} - \frac{5}{2}\frac{\Gamma_\perp^{sep}}{n}\right)}{\chi_e}\right)\right] \tag{24}$$

and

$$\gamma_i \equiv \frac{T_i}{T_e + T_i}\,,\ \gamma_e \equiv \frac{T_e}{T_e + T_i} \tag{25}$$

When atomic physics effects are not dominant, the leading order term for the pedestal width

$$\Delta_{TB}^{(o)} \equiv \frac{G}{b} = \frac{\ell n \left[\frac{16}{3} \, \frac{1}{2} \left(\frac{4\pi}{5\times 10^6 \mu_o} \right)^2 \left(\frac{B_\theta^2/2\mu_o}{p_{SoL}} \right) \frac{\left(1+\kappa^2\right)}{\varepsilon^2 g^2} \left(\frac{1}{2} \, \gamma_i \right)^{1/3} \left(\frac{\rho_i}{R} \right)^{2/3} - 1 \right]}{\left[\left(\frac{\Gamma_\perp^{sep}/n - \mathrm{V}_p}{D} \right) + \gamma_i \left(\frac{\frac{Q_{\perp i}^{sep}}{n \; T_i} - \frac{5}{2} \frac{\Gamma_\perp^{sep}}{n}}{\chi_i} \right) + \gamma_e \left(\frac{\frac{Q_{\perp e}^{sep}}{n \; T_e} - \frac{5}{2} \frac{\Gamma_\perp^{sep}}{n}}{\chi_e} \right) \right]} \tag{26}$$

is determined by the MHD pressure constraint and the edge transport physics. The atomic physics effects enter to higher order through the term $\left(\frac{aG}{b^2}\right)$ in Eq. (22).

7 Discussion

A model has been presented in which the edge pedestal parameters (gradient scale lengths and widths) depend not only on the local pedestal properties (plasma and impurity densities, temperatures, transport coefficients) but also on the particle and heat fluxes flowing across the pedestal from the core into the scrape-off layer, on the scrape-off layer pressure, on the penetration of neutrals recycling from the wall back into the pedestal, and on the maximum pedestal pressure and pressure gradients allowed for MHD stability. This model is represented by six nonlinear equations (Eqs. 1, 2, 3, 10, 14 and 16) which must be solved for the gradient scale lengths (L_n, L_{Te}, L_{Ti}) and the pedestal widths (Δ_n,Δ_{Te}, Δ_{Ti}). In order to evaluate the parameters in these equations, it is necessary to know 1) the transport coefficients ($D, \mathrm{V}_p, \chi_{i,e}$) in the edge pedestal, 2) the parameterization of the MHD function $\mathrm{A}_s(\mathrm{s})$ which defines the $s - \alpha$ diagram, 3) the particle and heat fluxes $\left(\Gamma_\perp^{sep}, \; Q_{\perp e,i}^{sep}\right)$ crossing the separatrix, 4) the density and temperatures at the separatrix$\left(n^{sep}, T_{i,e}^{sep}\right)$, and 5) the neutral and impurity densities in the edge pedestal. A calculation model has been developed [21] for the analysis of DIII-D that will provide $\left(\Gamma_\perp^{sep}, \; Q_{\perp i,e}^{sep}, \; n^{sep}, \; T_{i,e}^{sep} \text{ and } n_o^{ped}\right)$.

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