Plasma Waves, Heating and Current Drive

Andrew Seltzman and John-Patrick Floyd II
NRE4903 Special Problem, Fall 2006
Advisor: Prof. W. M. Stacey
December, 2006
(This paper is written to supplement the NRE6102 textbook Stacey, Fusion Plasma Physics)

Introduction

Modern tokamak design requires an in depth understanding of the theory and application behind the interaction of RF waves and the ionized particles in a plasma. Plasma wave interaction in a tokamak is normally divided into three main categories; RF heating, current drive, and plasma diagnostics.

In RF heating, wave energy is transferred to particle motion exploiting the wave-particle resonances in a plasma, selectively heating electrons or ions. By providing external heating the Hugill disruption limit (Stacey fig. 18.1) may be extended allowing for higher densities and fusion rates.

Rf current drive transfers organized momentum to the plasma in order to induce toroidal currents that generate the poloidal containment field. In modern tokamaks, the requirement of sustaining continuous currents without continuously increasing the current through the central solenoid, can be accomplished by using RF waves to induce currents in the plasma. This can be accomplished by landau damping or by selectively heating electrons or ions traveling an a specific direction, reducing their cross section and inducing a net current.

The temperature and density of a given plasma can be determined by probing the plasma with RF waves, and subsequently determining resonances, cutoffs and phase shift, or by passively observing the emitted RF radiation due to oscillations in the plasma.

While it is possible to solve the standard problems of plasma physics with the boxed equations, a thorough understanding of the subject requires the understanding of the basic physics behind the interactions between electromagnetic waves and particles. Such an understanding can be gained by working the derivations to gain insight into the fundamental nature of plasma waves and understanding the limits of resonance and cutoff for each type of wave. Such derivations and analysis are covered in the following paper.

Contents

I. Waves in an unmagnetized plasma.

a) <u>Electromagnetic Waves</u>

From Maxwell's Equations we have the following curl identities:

(1.1) $$\nabla\times B=\mu_0 j+\frac{1}{c^2}\frac{\partial E}{\partial t}$$

(1.2) $$\nabla\times E=-\frac{\partial B}{\partial t}$$

Differentiating on both sides of the magnetic field curl equation:

(1.3) $$\frac{\partial}{\partial t}\left[\nabla\times B\right]=\nabla\times\frac{\partial B}{\partial t}=\mu_0\frac{\partial j}{\partial t}+\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

Substituting in $\frac{\partial B}{\partial t}$ from the electric field curl equation:

(1.4) $$-\nabla\times\nabla\times E=\mu_0\frac{\partial j}{\partial t}+\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

Using the double curl identity $\nabla\times\nabla\times E=\nabla^2 E-\nabla\left(\nabla\bullet E\right)$

(1.5) $$\nabla\left(\nabla\bullet E\right)-\nabla^2 E=-\mu_0\frac{\partial j}{\partial t}-\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

Defining current generated by a species in a plasma as:

(1.6) $$j=\rho_\sigma v_\sigma=\sum_\sigma n_\sigma e_\sigma v_\sigma$$

The plasma **force balance equation** is then used to solve for v_σ

(1.7) $$m_\sigma n_\sigma\left(\frac{\partial}{\partial t}+v_\sigma\bullet\nabla\right)v_\sigma=-\nabla p_\sigma+n_\sigma e_\sigma\left(E+v_\sigma\times B\right)$$

Using the **cold plasma approximation** there are **no net particle flows**, since electrons scatter off ions and travel around them with out imparting significant momentum, we consider there to be **no pressure gradients**. Since the plasma is unmagnetized, there is **no Lorenz force**.

(1.8) $$m_\sigma n_\sigma\left(\frac{\partial}{\partial t}+\cancel{v_\sigma\bullet\nabla}\right)v_\sigma=-\cancel{\nabla p_\sigma}+n_\sigma e_\sigma\left(E+v_\sigma\times\cancel{B}\right)$$

The force balance equation reduces to

(1.9) $$\frac{\partial v_\sigma}{\partial t}=\frac{e_\sigma}{m_\sigma}E$$

Now $\frac{\partial j}{\partial t}$ can be found from the force balance

(1.10) $$\frac{\partial j}{\partial t}=\sum_\sigma n_\sigma e_\sigma\frac{\partial v_\sigma}{\partial t}=\sum_\sigma\frac{n_\sigma e_\sigma^2}{m_\sigma}E$$

Plugging $\frac{\partial j}{\partial t}$ into the dispersion relation

(1.11) $$\nabla(\nabla \bullet E) - \nabla^2 E = -\mu_0 \left(\sum_\sigma \frac{n_\sigma e_\sigma^2}{m_\sigma} E \right) - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$

Assuming waves of the form

(1.12) $$\exp\left[i(k \bullet r - \omega t) \right]$$

The Helmholtz wave equation can be used to specify

(1.13) $$\nabla^2 E = -k^2 E$$

(1.14) $$\nabla(\nabla \bullet E) = -k(k \bullet E)$$

and solving for

(1.15) $$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E$$

The general **form of the plasma dispersion relation** is found to be

(1.16) $$-k(k \bullet E) + k^2 E = \frac{\omega^2}{c^2} E - \mu_0 \left(\sum_\sigma \frac{n_\sigma e_\sigma^2}{m_\sigma} E \right)$$

Simplifying the dispersion relation for several cases

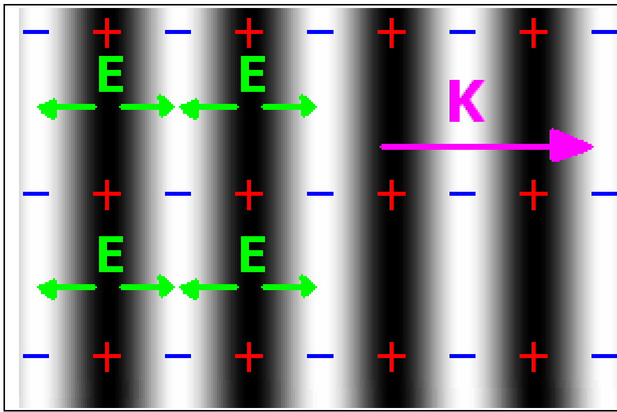
<u>For a longitudinal wave</u>, $k \parallel E$

(1.17) $$k(k \bullet E) = k^2 E$$

(1.18) $$\frac{\omega^2}{c^2} E = \mu_0 \left(\sum_\sigma \frac{n_\sigma e_\sigma^2}{m_\sigma} E \right)$$

(1.19) $$\omega^2 = \frac{1}{\varepsilon_0} \left(\sum_\sigma \frac{n_\sigma e_\sigma^2}{m_\sigma} \right) \equiv \omega_p^2$$

A longitudinal plasma wave **oscillates at the fundamental electron plasma frequency** and propagates by the electrostatic interactions of the plasma electrons. The longitudinal wave is **carried by electron density fluctuations** and is at higher frequency then the ion sound wave. In this limit, the ions appear virtually stationary to the electrons, and no pressure gradients are induced. The longitudinal plasma wave has **no resonances or cutoffs.**



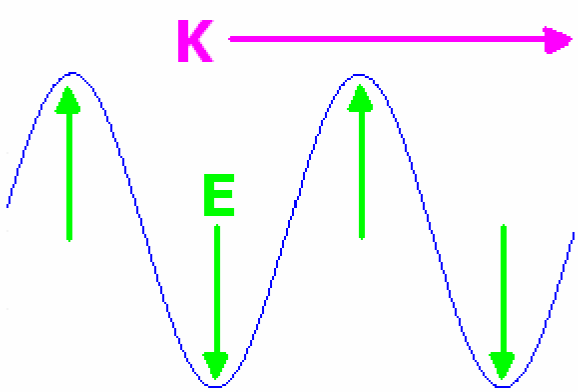
<u>For a transverse wave</u>, $k \perp E$

(1.20) $$-k\left(\cancel{k \cdot E}\right) + k^2 E = \frac{\omega^2}{c^2} E - \mu_0 \left(\sum_\sigma \frac{n_\sigma e_\sigma^2}{m_\sigma} E \right)$$

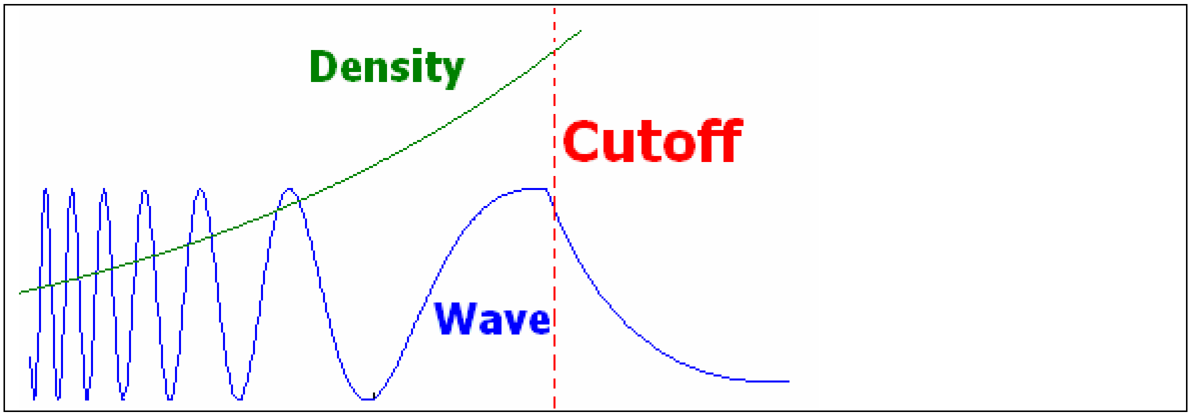
(1.21) $$c^2 k^2 = \omega^2 - \frac{1}{\varepsilon_0} \left(\sum_\sigma \frac{n_\sigma e_\sigma^2}{m_\sigma} \right)$$

(1.22) $$\omega^2 = c^2 k^2 + \omega_p^2$$

A transverse electromagnetic wave propagates through a plasma like a light wave through free space, however the presence of the plasma imposed a correction on wavelength based on plasma density.



The transverse plasma wave has **no resonances but has a cutoff when the wave frequency is below the plasma frequency.** As the wave propagates into higher density region, the wavelength increases until propagation vector $k = 2\pi / \lambda$ is forced to zero, thereby reflecting the wave back out of the plasma. If the cutoff region is sufficiently thin, it is possible that part of the wave will evanescently couple through the cutoff region and resume in an area of lower density, however, part of the wave will still be reflected.



b) Ion sound waves

To find the effect of propagating pressure waves in a plasma, we take waves to be of the form

(1.23) $$\exp\left[i\left(k\bullet r-\omega t\right)\right]$$

Expanding the position variable about its equilibrium position

(1.24) $$x\left(r,t\right)=x_0\left(r\right)+x_1\left(r,t\right)$$

The pressure balance equation now is approximated as

(1.25) $$m_in_i\left(\frac{\partial}{\partial t}v_i+v_i\bullet\nabla v_i\right)=n_ieE-\nabla p_i$$

Taking

(1.26) $$v_i\left(r,t\right)=v_0+v_1=v_0+A\exp\left[i\left(k\bullet r-wt\right)\right]$$

Using the **cold plasma approximation** there are **no net particle flows**, however in this case the ions carry the momentum of the wave and oscillate generating pressure gradients. Using poisons equation to equate E to the potential

(1.27) $$m_in_i\left(\frac{\partial}{\partial t}v_i+\cancel{v_i\bullet\nabla v_i}\right)=-n_ie\nabla\phi-\nabla p_i$$

(1.28) $$-i\omega m_in_{i0}v_{i1}=-n_{i0}eik\phi-ikp_{i1}$$

Using the adiabatic gas relation

(1.29) $$\frac{p}{p_0}=\gamma\frac{n}{n_0}$$

(1.30) $$p_{i1}=\gamma_in_{i1}\frac{p_0}{n_0}\Rightarrow\frac{p_0}{n_0}=kT=T_J$$

The pressure balance equation is now

(1.31)	$-i\omega m_i n_{i0} v_{i1} = -n_{i0} eik\phi - \gamma_i T_i ikn_{i1}$

Solving the pressure equation, we need to know potential as a function of density and temperature.
For electrons assume a Boltzman distribution

(1.32) $$n_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right)$$

Using he Taylor expansion for exp()

(1.33) $$n_e = n_{e0}\left(1 + \frac{e\phi_1}{T_e}\right)$$

(1.34) $$n_{e1} = n_{e0}\left(\frac{e\phi_1}{T_e}\right)$$

From Poisson's equation

(1.35) $$\varepsilon_0 \nabla \bullet E = \rho = -\varepsilon_0 \left(\nabla \bullet ik\phi_1\right) = \varepsilon_0 k^2 \phi_1$$

(1.36) $$\varepsilon_0 k^2 \phi_1 = \rho = e\left(n_{i1} - n_{e1}\right)$$

Substituting in electron density

(1.37) $$\frac{\varepsilon_0 k^2 \phi_1}{e} = n_{i1} - n_{e0}\left(\frac{e\phi_1}{T_e}\right)$$

and solving for n_{i1}

(1.38) $$n_{i1} = \left[n_{e0}\left(\frac{e}{T_e}\right) + \frac{\varepsilon_0 k^2}{e}\right]\phi_1$$

Solving the ion continuity equation

(1.39) $$\frac{\partial n_i}{\partial t} + \nabla \bullet \left(n_i v_i\right) = 0$$

and linearizing by removing higher order terms

(1.40) $$\frac{\partial n_i}{\partial t} + \nabla \bullet \left(n_{i0} v_{i1} + \cancel{n_{i1} v_{i1}}\right) = 0$$

(1.41) $$\frac{\partial n_i}{\partial t} + n_{i0} \nabla \bullet v_{i1} + v_{i0} \cancel{\nabla \bullet n_{i1}} = 0 \text{ v}$$

(1.42) $$i\omega n_1 = n_0 ikv_1$$

Now substituting in the expressions for second order terms and canceling out like terms

(1.43) $$-\cancel{i}\omega m_i \cancel{n_{i0}} \frac{\cancel{i}\omega \cancel{n_{i1}}}{\cancel{n_{i0}}\cancel{i}k} = -\frac{en_{i0}\cancel{i}k\cancel{n_{i1}}}{\left[n_{e0}\left(\frac{e}{T_e}\right) + \frac{\varepsilon_0 k^2}{e}\right]} - \gamma_i T_i \cancel{i}k\cancel{n_{i1}}$$

(1.44) $$\frac{\omega^2}{k^2} = \frac{en_{i0}/m_i}{\left[n_{e0}\left(\frac{e}{T_e}\right) + \frac{\varepsilon_0 k^2}{e} \right]} + \frac{\gamma_i T_i}{m_i}$$

(1.45) $$\left(\frac{\omega}{k}\right)^2 = \frac{T_e/m_i}{\left[1 + k^2 \frac{\varepsilon_0 T_e}{n_{i0} e^2} \right]} + \frac{\gamma_i T_i}{m_i}$$

Ion sound wave dispersion relation

(1.46) $$\left(\frac{\omega}{k}\right)^2 = \frac{T_e/m_i}{\left[1 + k^2 \lambda_d \right]} + \frac{\gamma_i T_i}{m_i}$$

(1.47) $$\omega^2 = k^2 \left[\frac{T_e/m_i}{\left[1 + k^2 \lambda_d \right]} + \frac{\gamma_i T_i}{m_i} \right] = k^2 v_s^2$$ where **Vs is the ion sound speed**

The ion sound wave is a longitudinal plasma wave that travels at the ion sound speed Vs by means of electrostatic interactions between plasma ions. The ion sound wave exists at much lower frequency then the longitudinal plasma wave. In this limit electrons move around ions much faster then the wave speed, thereby reestablishing electrostatic equilibrium. The ion sound wave travels by pressure waves, with ions contributing the majority of available mass and momentum. The ion sound wave has **no resonances however at sufficiently high frequencies ion sound waves will not propagate** due to the ions high mass.

II. Waves in a uniformly magnetized plasma

I. Electromagnetic waves

With a uniform magnetic field directed along the Z axis the pressure balance equation is

(2.1) $$m_\sigma n_\sigma \left(\frac{\partial}{\partial t} + \cancel{v_\sigma \bullet \nabla} \right) v_\sigma = -\cancel{\nabla p_\sigma} + n_\sigma e_\sigma \left(E + v_\sigma \times B \right)$$

(2.2) $$m_\sigma \frac{\partial v_\sigma}{\partial t} = e_\sigma \left(E + v_\sigma \times B \right)$$

(2.3) $$\frac{e_\sigma}{m_\sigma} E = -i\omega v_\sigma - \frac{e_\sigma}{m_\sigma} v_\sigma \times B$$

Defining

(2.4) $$\vec{B} = B\hat{z} \Rightarrow B_{\mathbf{U}} = B\delta_{\mathbf{UZ}}$$

(2.5) $$\Omega_\sigma = \frac{eB}{m_\sigma}$$

Using Einstein implicit summation, a general form of the wave equations may be found. Given that:

(2.6) $$[\mathrm{v}\times\mathrm{B}]_i = \varepsilon_{ijk} v_j B_k$$

(2.7) $$\delta_{ij} = \begin{cases} 1 \to i = j \\ 0 \to i \neq j \end{cases}$$

(2.8) $$\varepsilon_{ijk} = \begin{cases} 1 \to \text{ijk=xyz, yzx, zxy} \\ -1 \to \text{ijk=yxz, zyx, xzy} \\ 0 \to \text{for repeated index} \end{cases}$$

In a general form, using indices STU:

(2.9) $$\frac{e_\sigma}{m_\sigma} E_{\mathbf{S}} = -i\omega v_{\mathbf{S}} - \varepsilon_{\mathbf{STU}} \frac{e_\sigma}{m_\sigma} v_{\mathbf{T}} B_{\mathbf{U}} \delta_{\mathbf{UZ}}$$

The individual components may now be written as

(2.10) $$\frac{e_\sigma}{m_\sigma} E_x = -i\omega v_x - \frac{e_\sigma}{m_\sigma} v_y \times B_z = -i\omega v_x - \Omega v_y$$

(2.11) $$\frac{e_\sigma}{m_\sigma} E_y = -i\omega v_y + \frac{e_\sigma}{m_\sigma} v_x \times B_z = -i\omega v_y + \Omega v_x$$

(2.12) $$\frac{e_\sigma}{m_\sigma} E_z = -i\omega v_z$$

Solving for velocity components to find currents
Solving for Vy

(2.13) $$v_y = -\frac{\frac{e_\sigma}{m_\sigma} E_y - \Omega v_x}{i\omega}$$

And substituting it into the Vx relation

(2.14) $$-\frac{e_\sigma}{m_\sigma} E_x = i\omega v_x - \Omega \frac{\frac{e_\sigma}{m_\sigma} E_y - \Omega v_x}{i\omega}$$

(2.15) $$-i\omega v_x - \frac{\Omega^2 v_x}{i\omega} = \frac{e_\sigma}{m_\sigma} E_x - \Omega \frac{e_\sigma E_y}{m_\sigma i\omega}$$

(2.16) $$v_x \left[-i\omega - \frac{\Omega^2}{i\omega} \right] = \frac{e_\sigma}{m_\sigma} \left[E_x - \frac{\Omega E_y}{i\omega} \right]$$

(2.17) $$v_x = \frac{e_\sigma}{m_\sigma} \left(\frac{E_x - \frac{\Omega E_y}{i\omega}}{-i\omega - \frac{\Omega^2}{i\omega}} \right) \left(\frac{i\omega}{i\omega} \right) = \frac{e_\sigma}{m_\sigma} \left(\frac{i\omega E_x - \Omega E_y}{\omega^2 - \Omega^2} \right)$$

Now solving for jx

Current Density

(2.18) $$j_x = \sum_\sigma n_\sigma e_\sigma v_\sigma = \sum_\sigma \frac{ne_\sigma^2}{m_\sigma}\left(\frac{i\omega E_x - \Omega E_y}{\omega^2 - \Omega^2}\right)$$

Likewise

(2.19) $$j_y = \sum_\sigma n_\sigma e_\sigma v_\sigma = \sum_\sigma \frac{ne_\sigma^2}{m_\sigma}\left(\frac{i\omega E_y - \Omega E_x}{\omega^2 - \Omega^2}\right)$$

(2.20) $$j_z = \sum_\sigma n_\sigma e_\sigma v_\sigma = \sum_\sigma \frac{ne_\sigma^2}{m_\sigma}\frac{E_z}{i\omega}$$

Knowing j, the dispersion relation can be solved from Maxwell's equations

(2.21) $$\nabla(\nabla\bullet E) - \nabla^2 E = -\mu_0\frac{\partial j}{\partial t} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

(2.22) $$-k(k\bullet E) + k^2 E = \frac{\omega^2}{c^2}E - \mu_0\frac{\partial}{\partial t}\left[\sum_\sigma \frac{ne_\sigma^2}{m_\sigma}\left(\frac{i\omega E_x - \Omega E_y}{\omega^2 - \Omega^2}\right)\right]$$

(2.23) $$\mu_0\sum_\sigma \frac{ne_\sigma^2}{m_\sigma}\left(\frac{\omega^2 E_x + i\omega\Omega_\sigma E_y}{\omega^2 - \Omega_\sigma^2}\right) - \frac{\omega^2}{c^2}E = k(k\bullet E) - k^2 E$$

(2.24) $$\frac{1}{c^2}\sum_\sigma \frac{ne_\sigma^2}{\varepsilon_0 m_\sigma}\left(\frac{\omega^2}{\omega^2 - \Omega_\sigma^2}E_x + \frac{i\omega\Omega_\sigma}{\omega^2 - \Omega_\sigma^2}E_y\right) - \frac{\omega^2}{c^2}E = k(k\bullet E) - k^2 E$$

(2.25) $$\sum_\sigma \frac{ne_\sigma^2}{\varepsilon_0 m_\sigma}\left(\frac{\omega^2}{\omega^2 - \Omega_\sigma^2}E_x + \frac{i\omega\Omega_\sigma}{\omega^2 - \Omega_\sigma^2}E_y\right) - \omega^2 E = c^2\left[k(k\bullet E) - k^2 E\right]$$

(2.26) $$\sum_\sigma \frac{\omega_p^2\omega^2}{\omega^2 - \Omega_\sigma^2}E_x + i\sum_\sigma \frac{\omega_p^2\omega\Omega_\sigma}{\omega^2 - \Omega_\sigma^2}E_y - \omega^2 E = c^2\left[k(k\bullet E) - k^2 E\right]$$

(2.27) $$\left(c^2 k_z^2 - \omega^2 + \sum_\sigma \frac{\omega_p^2\omega^2}{\omega^2 - \Omega_\sigma^2}\right)E_x + i\left(\sum_\sigma \frac{\omega_p^2\omega\Omega_\sigma}{\omega^2 - \Omega_\sigma^2}\right)E_y - \left(c^2 k_x k_z\right)E_z = 0$$

Likewise for the other components

(2.28) $$-i\left(\sum_\sigma \frac{\omega_p^2\omega\Omega_\sigma}{\omega^2 - \Omega_\sigma^2}\right)E_x + \left(c^2 k^2 - \omega^2 + \sum_\sigma \frac{\omega_p^2\omega^2}{\omega^2 - \Omega_\sigma^2}\right)E_y = 0$$

(2.29) $$-\left(c^2 k_x k_z\right)E_x + \left(c^2 k_x^2 - \omega^2 + \sum_\sigma \omega_{p\sigma}^2\right)E_z = 0$$

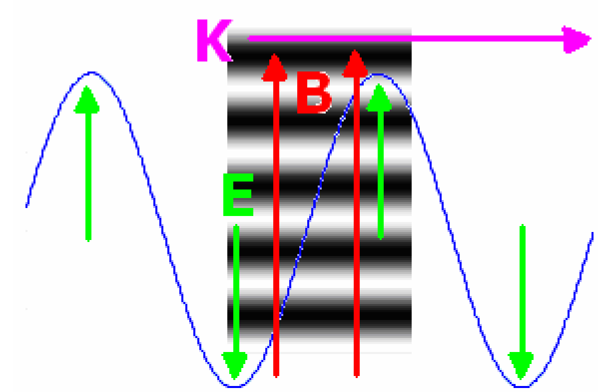
For waves with $k \perp B$, $E \parallel B$ and $k_x = k_y = 0$

(2.30) $$-\left(c^2 \not{k_x} k_z\right)E_x + \left(c^2 \not{k_x^2} - \omega^2 + \sum_\sigma \omega_{p\sigma}^2\right)E_z = 0$$

<u>O-mode resonance</u>

(2.31)	$\omega^2 = \sum_\sigma \omega_{p\sigma}^2$ **O-mode resonance for linearly polarized waves**

The O mode resonance occurs for a linearly polarized wave traveling perpendicular to the magnetic field with electric field parallel to the magnetic field. In this mode, the plasma ions or electrons are excited along the field line at their fundamental oscillation frequency. This excitation is exactly the same as in an unmagnetized plasma since the particle motion is parallel to the magnetic field, eliminating any Lorenz force interaction.



For waves propagating along B, $v \parallel B$ **and** $k_x = 0$

(2.32) $$\left(c^2 k_z^2 - \omega^2 + \sum_\sigma \frac{\omega_p^2 \omega^2}{\omega^2 - \Omega_\sigma^2}\right) E_x + i\left(\sum_\sigma \frac{\omega_p^2 \omega \Omega_\sigma}{\omega^2 - \Omega_\sigma^2}\right) E_y - \left(c^2 \cancel{k_x} k_z\right) E_z = 0$$

(2.33) $$\left(c^2 k_z^2 - \omega^2 + \sum_\sigma \frac{\omega_p^2 \omega^2}{\omega^2 - \Omega_\sigma^2}\right)^2 - \left(\sum_\sigma \frac{\omega_p^2 \omega \Omega_\sigma}{\omega^2 - \Omega_\sigma^2}\right)^2 = 0$$

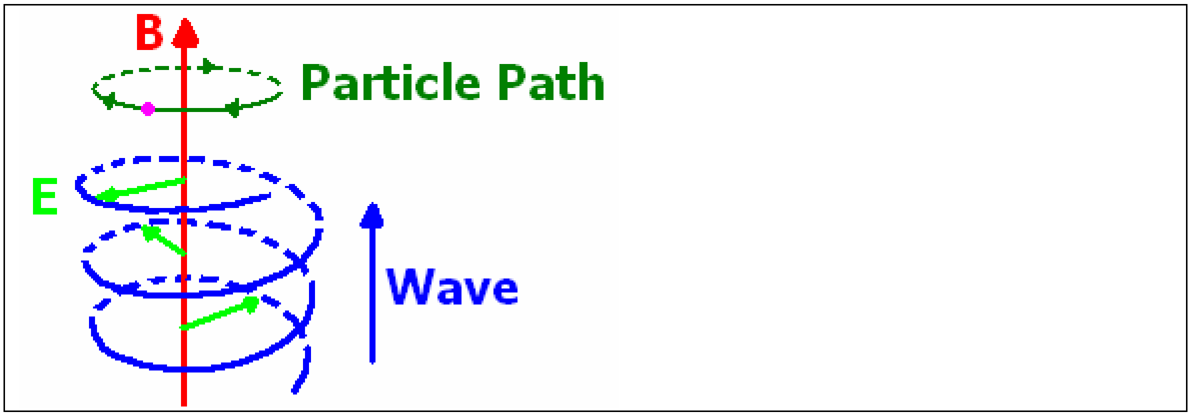
(2.34) $$c^2 k_z^2 - \omega^2 + \sum_\sigma \frac{\omega_p^2 \omega^2}{\omega^2 - \Omega_\sigma^2} \pm \sum_\sigma \frac{\omega_p^2 \omega \Omega_\sigma}{\omega^2 - \Omega_\sigma^2} = 0$$

(2.35) $$c^2 k_z^2 - \omega^2 + \sum_\sigma \frac{\omega_p^2 \omega}{\underbrace{\omega^2 - \Omega_\sigma^2}_{(\omega+\Omega_\sigma)(\omega-\Omega_\sigma)}} \left(\omega \pm \Omega_\sigma\right) = 0$$

<u>X-mode resonance</u>

(2.36)	$c^2 k_z^2 - \omega^2 + \sum_\sigma \frac{\omega_p^2 \omega}{(\omega \pm \Omega_\sigma)} = 0$ **X-mode resonance for CP wave**

The X mode resonance occurs for a circularly polarized wave traveling along the magnetic field. At resonance the rotating electric field accelerates the electrons or ions at their gyro frequency as they are guided in a circular path around the magnetic field lines..



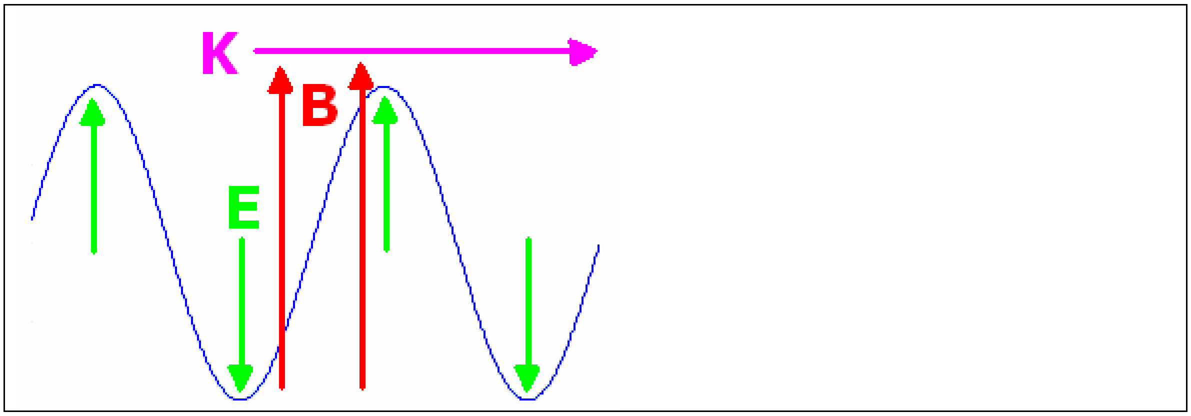
For $E_x = \pm i E_y$ this corresponds to **the x-mode cyclotron resonance** for a circularly polarized wave. The CP wave will penetrate the plasma for k>0 given by

(2.37) $$\omega^2 > \sum_\sigma \frac{\omega_p^2 \omega}{(\omega \pm \Omega_\sigma)}$$

For waves with $k \perp B$ and $E_x = E_y = 0$ the dispersion relations are

(2.38) $$\omega^2 = c^2 k_x^2 + \sum_\sigma \omega_{p\sigma}^2$$

Transverse wave with $k \perp B$ **and** $E_x = E_y = 0$



II. Hybrid Waves

Hybrid waves are combinations of longitudinal and transverse waves with $k \perp B$. They are a hybrid of two frequencies, one relating to the ion cyclotron frequency and one relating to the fundamental plasma frequency.

(2.39) $$c^2k^2 = \omega^2 - \sum_\sigma \frac{\omega_p^2\omega^2}{\omega^2-\Omega_\sigma^2} - \frac{\left(\sum_\sigma \frac{\omega_p^2\Omega_\sigma}{\omega^2-\Omega_\sigma^2}\right)^2}{\left(1-\sum_\sigma \frac{\omega_p^2}{\omega^2-\Omega_\sigma^2}\right)} = 0$$

Which has resonances at

(2.40) $$1-\sum_\sigma \frac{\omega_p^2}{\omega^2-\Omega_\sigma^2} = 0$$

The upper hybrid resonances are given by

(2.41) $$\omega^2 = \omega_{pe}^2 + \Omega_\sigma^2$$

and the lower hybrid resonances

(2.42) $$\omega^2 = \frac{\omega_{pi}^2\Omega_\sigma}{\omega_{pe}^2+\Omega_e^2}$$

If $\omega_{pe} \gg \Omega_e$ the lower hybrid resonance is approximately

(2.43) $$\omega_{LH}^2 \approx \left|\Omega_e\Omega_i\right|$$

III. Heating and Current Drive

a) Fast Ions

Neutral beam injection involves the shooting of a high energy beam of neutral atoms into the plasma to ionize the neutrals, and cause the resulting fast ions lose their energy to the plasma through kinetic and electromagnetic interaction.

i. Collisions with Plasma Electrons

The momentum balance between the fast ions and electrons, assuming a Maxwellian distribution, is

(3.1) $$n_b M_b \frac{d\langle \mathbf{V}\rangle}{dt} = -m_e \int \frac{d\langle \upsilon\rangle}{dt} f_e(\upsilon) d^3\upsilon$$

The equation of motion for the average electron velocity is

(3.2) $$\frac{d\langle \upsilon\rangle}{dt} = -\upsilon_{eb}(\upsilon - \mathbf{V})$$

where

(3.3) $$\upsilon_{eb} = \frac{n_b Z_b^2 e^4 \ln\Lambda}{4\pi\varepsilon_0^2 m_e^2 \left|\upsilon - \mathbf{V}\right|^3}$$

Substituting 41 into 39 yields 42

(3.4) $$\frac{d\langle \mathbf{V}\rangle}{dt} = \frac{Z_b^2 e^4 \ln\Lambda}{4\pi\varepsilon_0^2 m_e M_b}\int \frac{\upsilon - \mathbf{V}}{|\upsilon - \mathbf{V}|^3} f_e d^3\upsilon = -\frac{Z_b^2 e^4 \ln\Lambda}{4\pi\varepsilon_0^2 m_e M_b}\frac{\partial \mathbf{I}}{\partial \mathbf{V}}$$

with

(3.5) $$I(\mathbf{V}) = -\int \frac{f_e d^3\upsilon}{|\upsilon - \mathbf{V}|}$$

This integral is evaluated using a isotropic velocity space distribution to show

(3.6) $$\frac{d\langle \mathbf{V}\rangle}{dt} = -\frac{\sqrt{2} n_e Z_b^2 e^4 \sqrt{m_e} \ln\Lambda}{12\pi\sqrt{\pi}\varepsilon_0^2 M_b T_e^{\frac{3}{2}}}\mathbf{V}$$

Because the frictional drag of the electrons on the fast ions varies as $T_e^{-\frac{3}{2}}$, taking the scalar product of 45 with $M_b\mathbf{V}$ gives the fast ion energy reduction rate.

(3.7) $$\frac{dW_b}{dt} = -\frac{\sqrt{2} n_e Z_b^2 e^4 \sqrt{m_e} \ln\Lambda}{6\pi\sqrt{\pi}\varepsilon_0^2 M_b} W_b \equiv -\frac{W_b}{\tau_{eb}}$$

The same process is followed to find the rate at which fast ion energy is reduced due to plasma ions. These terms can be combined to show the slowing down of the fast ions by plasma electrons and ions (12.33)

(3.8) $$\frac{dW_b}{dt} = -\frac{\sqrt{2} n_e Z_b^2 e^4 \sqrt{m_e} \ln\Lambda}{6\pi\sqrt{\pi}\varepsilon_0^2 M_b}\left(\frac{W_b}{T_e^{\frac{3}{2}}} + \frac{C}{W_b^{\frac{1}{2}}}\right)$$

where

(3.9) $$C = \frac{3\pi^{\frac{1}{2}} Z M_b^{\frac{3}{2}}}{4 m_e^{\frac{1}{2}} m_i}$$

The two terms are equal when the fast ion energy has a "critical" value (12.34)

(3.10) $$\frac{W_{b,crit}}{T_e} = C^{\frac{2}{3}} \approx 19$$

b) Electromagnetic Waves

i. Wave Propagation

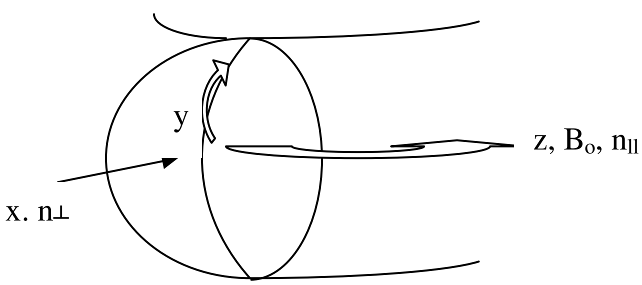
The plasma model based on the cold plasma approximation as detailed previously can be used to discuss in-plasma propagation of the electromagnetic waves used for heating and current drive. Based on Maxwell's equations and the usual exponential wave field representation, we arrive at a representation for the dielectric tensor

$$(3.11) \qquad -\mathbf{k}\times\mathbf{k}\times\mathbf{E} = \frac{\omega}{c^2}\left(\mathbf{E}+\frac{i\varepsilon_0}{\omega}\mathbf{j}\right) = \frac{\omega^2}{c^2}\vec{\varepsilon}\bullet\mathbf{E}$$

which is detailed in-depth in the section on the dispersion matrix representation. Using $\mu_0 c^2 = \varepsilon_0$, we can represent the refractive index as $n = [\mathbf{k}c/w]$, allowing the cold plasma dispersion relation to be written as

$$(3.12) \qquad \varepsilon_\perp n_\perp^4 - \left[(\varepsilon_\perp - n_{ll})(\varepsilon_\perp + \varepsilon_{ll}) + \varepsilon_{xy}^2\right]n_\perp^2 + \varepsilon_{ll}\left[(\varepsilon_\perp - n_{ll}^2)^2 + \varepsilon_{xy}^2\right] = 0$$

where B_0 is in the z-direction and x and y are normal to z (in a tokamak, x, y, and z refer to the radial, poloidal, and toroidal directions in a tokamak. The perpendicular and parallel refractive indexes are defined with respect to the z-direction of the equilibrium magnetic field, $n_{\perp/ll} = k_{\perp/llc/\omega}$



The wave will only propagate in the plasma (and therefore be useful for heating and current drive) if $n_\perp^2 > 0$. Using this, we solve the dispersion relation for $n_\perp^2$ at various radii of the plasma using the local parameter values. For certain values of plasma parameters density and magnetic field, and the wave frequency being injected, $n_\perp \to \infty$, indicating a wave frequency absorption by the plasma. At other values, $n_\perp \to 0$, indicating a cutoff region where the wave is reflected. Beyond the cutoff surface is the evanescent region where the wave will decay exponentially with radial position. However, if conditions support wave propagation beyond the evanescent region, the wave will "tunnel through" the region, and continue propagating.

ii. <u>Wave Heating Physics</u>

Resonance frequencies, corresponding with the frequencies found above in Eqs. 31, 36, and 37 include the Ion Cyclotron resonance (ICRH), the Lower Hybrid resonance(LH), and the Electron Cyclotron resonance (ECRH).

The ICRH is the ion-ion resonance with wave frequencies in the 30 MHz to 120 MHz range. A density limit makes the outer plasma regions evanescent for the ICRH wave, but upon "tunneling through" the evanescent region, it propagates well in the plasma interior. The Lower Hybrid is the combination ion and electron cyclotron frequency resonance with wave frequencies in the 1 GHz to 8 GHz range. The ECRH resonance is the upper hybrid frequency with frequencies in the 100 GHz to 200 GHz range. The ECRH wave has complicated requirements for propagation dependent on factors including the side from which the wave is launched and the mode of the wave, either X or O.

iii. <u>Current Drive</u>

While LH waves are not very successful at plasma heating, they have proven excellent at driving plasma current. In order to drive this current, the LH waves are launched with a well defined phase velocity along the magnetic field chosen to resonate with 100 keV electrons. When the electrons' energy is increased by absorbing wave energy via Landau damping, they become less collisional and lose momentum at a reduced rate. This drives current because the electrons are no longer in equilibrium because of the lower momentum loss. The same effect occurs in electrons heated by the ECRH wave, but with a lower effectiveness. A figure of merit for current drive efficiency is

(3.13) $$\gamma_{CD} = \frac{RI}{P}\frac{\bar{n}_e}{10^{20}}\left(m^{-2}AW^{-1}\right)$$

So far, LH current drive is the most efficient method.

IV. <u>Dispersion matrix for a magnetized plasma</u>

The current densities in a plasma can be represented in an alternate notation by using Ohm's law to represent the current densities in matrix notation.

(4.1) $$j = qvn = \sigma E$$

(4.2) $$\sigma = \frac{j}{E}$$

In a magnetized plasma with $\vec{B} = B\hat{z}$:

(4.3) $$j_x = \sum_\sigma n_\sigma e_\sigma v_\sigma = \sum_\sigma \frac{ne_\sigma^2}{m_\sigma}\left(\frac{i\omega E_x - \Omega E_y}{\omega^2 - \Omega^2}\right)$$

(4.4) $$j_y = \sum_\sigma n_\sigma e_\sigma v_\sigma = \sum_\sigma \frac{ne_\sigma^2}{m_\sigma}\left(\frac{i\omega E_y - \Omega E_x}{\omega^2 - \Omega^2}\right)$$

(4.5) $$j_z = \sum_\sigma n_\sigma e_\sigma v_\sigma = \sum_\sigma \frac{ne_\sigma^2}{m_\sigma}\frac{E_z}{i\omega}$$

Splitting up the x,y,z components of the electric field we can represent the current in the form of the **conductivity tensor**. Note that the X and Y components of the current densities(orthogonal to the magnetic field) represent the cyclotron motion, while the Z component(parallel to the magnetic field) represents linear oscillations at the fundamental plasma frequency.

(4.6) $$j = \underbrace{\begin{bmatrix} \frac{ne_\sigma^2}{m_\sigma}\frac{i\omega}{\omega^2-\Omega^2} & -\frac{ne_\sigma^2}{m_\sigma}\frac{\Omega}{\omega^2-\Omega^2} & 0 \\ \frac{ne_\sigma^2}{m_\sigma}\frac{\Omega}{\omega^2-\Omega^2} & \frac{ne_\sigma^2}{m_\sigma}\frac{i\omega}{\omega^2-\Omega^2} & 0 \\ 0 & 0 & \frac{ine_\sigma^2}{\omega m_\sigma} \end{bmatrix}}_{\sigma} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

This can be modified into the dielectric tensor using the definition of susceptibility

(4.7) $$\chi = \left[1/\left(-i\omega\varepsilon_0\right)\right]\sigma$$

and that of the dielectric constant

(4.8) $$\varepsilon = 1 + \chi$$

(4.9) $$\varepsilon = 1 - \frac{\sigma}{i\omega\varepsilon_0}$$

the dielectric tensor can be written as

(4.10) $$\varepsilon = \begin{bmatrix} 1-\frac{ne_\sigma^2}{\varepsilon_0 m_\sigma}\frac{1}{\omega^2-\Omega^2} & \frac{ne_\sigma^2}{\varepsilon_0 m_\sigma}\frac{1}{i\omega}\frac{\Omega}{\omega^2-\Omega^2} & 0 \\ -\frac{ne_\sigma^2}{\varepsilon_0 m_\sigma}\frac{1}{i\omega}\frac{\Omega}{\omega^2-\Omega^2} & 1-\frac{ne_\sigma^2}{\varepsilon_0 m_\sigma}\frac{1}{\omega^2-\Omega^2} & 0 \\ 0 & 0 & 1-\frac{1}{\omega^2}\frac{ne_\sigma^2}{\varepsilon_0 m_\sigma} \end{bmatrix}$$

Defining the fundamental plasma frequency as

(4.11) $$\omega_{p\sigma}^2 = \frac{e_\sigma^2 n_\sigma}{\varepsilon_0 m_\sigma}$$

(4.12)
$$\varepsilon = \begin{bmatrix} 1-\frac{\omega_{p\sigma}^2}{\omega^2-\Omega^2} & \frac{\Omega}{i\omega}\frac{\omega_{p\sigma}^2}{\omega^2-\Omega^2} & 0 \\ -\frac{\Omega}{i\omega}\frac{\omega_{p\sigma}^2}{\omega^2-\Omega^2} & 1-\frac{\omega_{p\sigma}^2}{\omega^2-\Omega^2} & 0 \\ 0 & 0 & 1-\frac{\omega_{p\sigma}^2}{\omega^2} \end{bmatrix}$$

Defining

(4.13)
$$S = 1-\sum_{\sigma}\frac{\omega_{p\sigma}^2}{\omega^2-\Omega^2}$$

(4.14)
$$D = \sum_{\sigma}\frac{\Omega}{\omega}\frac{\omega_{p\sigma}^2}{\omega^2-\Omega^2}$$

(4.15)
$$P = 1-\sum_{\sigma}\frac{\omega_{p\sigma}^2}{\omega^2}$$

The **dielectric tensor** (Stacey Sect. 12.4.1.1) can now be simplified to

(4.16)
$$\varepsilon = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}$$

where S and D stand for sum and difference of the R and L hand circular polarizations

(4.17)
$$S = \frac{1}{2}(R+L)$$

(4.18)
$$D = \frac{1}{2}(R-L)$$

(4.19)
$$R = 1-\sum_{\sigma}\frac{\omega_{p\sigma}^2}{\omega(\omega+\Omega)}$$

(4.20)
$$L = 1-\sum_{\sigma}\frac{\omega_{p\sigma}^2}{\omega(\omega-\Omega)}$$

The cold dispersion relation is written in terms of the refractive index terms

(2.64)
$$N_{ll} = \frac{k_{ll}}{k_0}; N_{\perp} = \frac{-i}{k_0}\frac{d}{dx}$$

The plasma is assumed homogenous in the parallel direction.

$$\begin{pmatrix} S - N_{ll}^2 & iD & N_\perp N_{ll} \\ iD & S - N_{ll}^2 - N_\perp^2 & 0 \\ N_\perp N_{ll} & 0 & P - N_\perp^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

Assuming the plasma conductivity is infinite in the parallel direction, or taking the zero electron mass limit implies that the parallel electric field cannot penetrate the plasma. Taking this limit in 69 we arrive at

$$\begin{pmatrix} k_0^2 S - k_{ll}^2 & -ik_0^2 D \\ ik_0^2 D & k_0^2 S - k_{ll}^2 + \frac{d^2}{dx^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = 0$$

The other components of the fast wave field follow from Maxwell's equation $i\omega \mathbf{B} = \nabla \times \mathbf{E}$. Eliminating E_x from 12, we obtain the fast wave equation 71 and 72

$$\frac{d^2 E_y}{dx^2} + k_{\perp FW} E_y = 0$$

$$k_{\perp FW}^2 = k_0^2 N_{\perp FW}^2 = (k_0^2 S - k_{ll}^2) - \frac{(k_0^2 D)^2}{(k_0^2 S - k_{..}^2)}$$

V. Antenna Arrays

c) Use of arrays

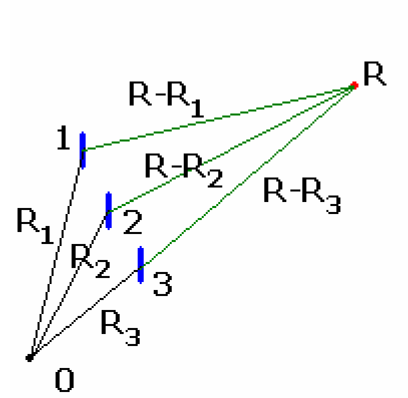
In a tokamak Plasma is heated by launching an RF wave into the plasma at the given polarization, direction, and frequency. The RF wave is transferred to the tokamak from a generator using coaxial cable or waveguides, however to launch the wave, an antenna must be used to effectively couple the cable or waveguide to free space or the plasma within the reactor.

For high frequencies, a feed horn is used at the end of the waveguide to launch and direct the wave. While the horn antenna is capable of easily directing an RF wave at an angle, lower frequency antennas such as a dipole or inductive strap(loop) radiate omni directionally, preventing efficient current drive.

A strap antenna, is positioned near the surface of the first wall of the tokamak, near the plasma vacuum interface, however this type of antenna would normally radiate omni directionally. At any given point in space, the phases of the RF wave emitted from each element in the antenna array will interfere, either constructively or destructively, governing the intensity of the RF power radiated in a given direction. By properly adjusting the spacing and drive phases of the individual elements, the array can launch an RF wave at any required angle.

d) Array factor calculations

For a set of elements with spacing |R-Rn|



(5.1) $$E_n(r,\theta,\phi) = E'(\theta,\phi)\frac{e^{-jk|R-R_n|}}{|R-R_n|}$$

(5.2) $$E_n(r,\theta,\phi) = E'(\theta,\phi)\frac{e^{-jk(r-r'\cos(\psi))}}{r}$$

For a given element:

(5.3) $$E_n(r,\theta,\phi) = E'(r,\theta,\phi)\,e^{jkr_n'\cos(\psi_n)}$$

Summing over all elements

(5.4) $$E_n(r,\theta,\phi) = E'(r,\theta,\phi)\sum_n a_n e^{jkr_n'\cos(\psi_n)}$$

where $E'(r,\theta,\phi)$ is the radiation pattern of a single element.

(5.5) $$E_n(r,\theta,\phi) = E'(r,\theta,\phi)\sum_n a_n e^{-jn(kd\cos(\theta)-\zeta)}$$

where

$k = \frac{2\pi}{\lambda}$

ζ = phase shift between elements

d= element spacing

θ = polar angle about array length

(5.6) $$AF = \sum_n a_n e^{-jn(kd\cos(\theta)-\zeta)}$$

Using the identity

(5.7) $$\sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x}$$

(5.8) $$AF = a_n \frac{1-e^{jN(kd\cos(\theta)-\zeta)}}{1-e^{j(kd\cos(\theta)-\zeta)}}$$

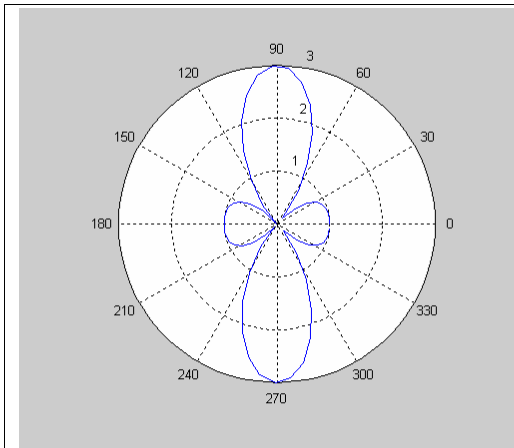
(5.9) $$|AF| = |a_n| \left| \frac{e^{j\frac{N}{2}(kd\cos(\theta)-\zeta)}}{e^{j\frac{1}{2}(kd\cos(\theta)-\zeta)}} \right| \left| \frac{e^{-j\frac{N}{2}(kd\cos(\theta)-\zeta)} - e^{j\frac{N}{2}(kd\cos(\theta)-\zeta)}}{e^{-j\frac{1}{2}(kd\cos(\theta)-\zeta)} - e^{j\frac{1}{2}(kd\cos(\theta)-\zeta)}} \right|$$

Array factor magnitude

(5.10) $$|AF| = a_0 \left| \frac{\sin\left[\frac{N}{2}\left(kd\cos(\theta)-\zeta\right)\right]}{\sin\left[\frac{1}{2}\left(kd\cos(\theta)-\zeta\right)\right]} \right|$$

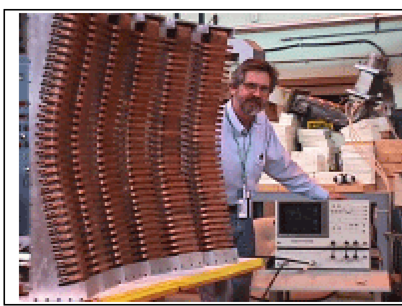
$\zeta = 0$ for broadside array, let $kd = \pi$ for $d = \frac{\lambda}{2}$

(5.11) $$|AF| = a_0 \left| \frac{\sin\left[\frac{3}{2}\pi\cos(\theta)\right]}{\sin\left[\frac{1}{2}\pi\cos(\theta)\right]} \right|$$



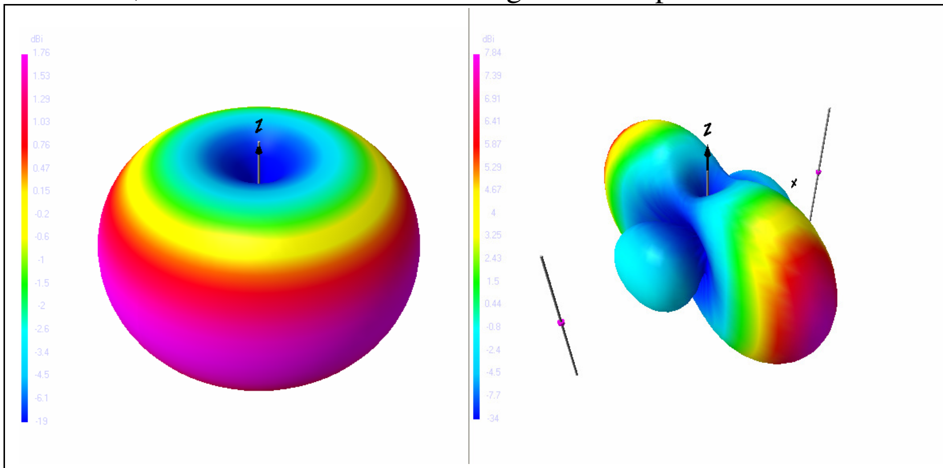
Azimuthal array factor pattern

The radiation pattern generated is the product of the array factor and the individual element pattern.



NTSX strap antenna array; 6 elements (Courtesy of Dept of Phys, Princeton University)

In this case a set of 3 strap antennas on the wall of a tokamak can direct the RF into a narrower beam pointing directly inward, or by adjusting the phases of the array elements, can direct the beam at an angle with respect to the wall.



Single element pattern vs array pattern

VI. Ion Cyclotron, Lower Hybrid and Alfven Wave Heating Methods

a) Ion Cyclotron Heating

i. Introduction

Collisional frequencies of plasma are small, and for a JET type machine, the gyration time for a cyclotron gyration is very short. The gyroradii of the ions and electrons is small compared to the plasma size. This implies that the plasma is nearly non-collisional

Parameter	
Electron coll. Freq. v_e	10kHz
Ion coll. Freq. v_i	100 Hz
Parameter Value in JET-like plasma	
R_0	3 m
a_p	1.5 m
Electron gyroradius	.05 mm
Ion gyroradius	3 mm
Electron MFP/toroidal rev.	3 km/150
Ion MFP/ toroidal rev.	5

	km/250

Typical ICH system parameters	
Frequency	10-100 MHz
Power	2 MW/strap
Voltage	10-50 kV
Central Conductor dimensions	W=.2 m, L=1 m, dist. to plasma/wall=5 cm/20 cm

Typical ICH system parameters are shown above. ICH antennas are often built as boxes containing several "central conductor" straps to which the voltage is applied.

ii. Linearity

The RF only causes a small perturbation of the particle trajectory because the RF magnetic field is much smaller than the static field, and the electric field is much smaller than the **v**x**b** field associated with charged particle thermal motion. The perturbation of the parallel motion is also small, as shown below.

The equation of motion of a particle in a RF field, $\mathbf{v}=\mathbf{v_0}+\mathbf{v}$ (thermal plus perturbed)

(6.1) $$m\left(\frac{d\mathbf{v}_0}{dt}+\frac{d\mathbf{v}}{dt}\right)=Ze(\mathbf{E}+\mathbf{v}_0\times\mathbf{B}_0+\mathbf{v}_0\times\mathbf{B}+\mathbf{v}\times\mathbf{B}_0+\mathbf{v}\times\mathbf{B})$$

subtracting the unperturbed part describing the unperturbed cyclotron motion

(6.2) $$m\left(\frac{d\mathbf{v}_0}{dt}\right)=Ze\mathbf{v}_0\times\mathbf{B}_0$$

leaves the perturbed equation of motion

(6.3) $$m\left(\frac{d\mathbf{v}}{dt}\right)=Ze(\mathbf{E}+\mathbf{v}_0\times\mathbf{B}+\mathbf{v}\times\mathbf{B}_0+\mathbf{v}\times\mathbf{B})$$

assuming the **v**x**B** term is negligible, we arrive at a equation linear in the perturbed field amplitude

(6.4) $$m\left(\frac{d\mathbf{v}}{dt}\right)=Ze(\mathbf{E}+\mathbf{v}_0\times\mathbf{B}+\mathbf{v}\times\mathbf{B}_0)$$

We can now estimate the correction to the parallel uniform motion due to the RF field. Taking the parallel component of 3 and $\frac{d}{dt}=\omega_{ci}$, for an ion we get

(6.5) $$m\omega_c v_{ll} = Ze\left[E_{ll} + \left(\mathbf{v}_0 \times \mathbf{B}\right)_{ll}\right] ==> v_{ll} \approx \left|\frac{E_{ll}}{B_0}\right| \approx V_{ti}\left|\frac{B_{RF}}{B_0}\right|$$

this implies that the RF-induced perturbation is small compared to the thermal ion velocity, which implies that linearization is justified.

Although 4 is linear in the fields, it is not for **r** and **v**. For **r**, we see that the RF fields causes little perturbation in the particle trajectories, enabling us to neglect the perturbations, and write

(6.6) $$\mathbf{E}(\mathbf{r}) \approx \mathbf{E}(\mathbf{r}_0), \mathbf{B}(\mathbf{r}) \approx \mathbf{B}(\mathbf{r}_0)$$

this implies that the equation governing the velocity perturbation is

(6.7) $$m\left(\frac{d\mathbf{v}}{dt}\right) = Ze(\mathbf{v}_0 \times \mathbf{B}(\mathbf{r}_0) + \mathbf{v} \times \mathbf{B}_0 + \mathbf{E}(\mathbf{r}_0))$$

which is a linear equation that can be solved explicitly. By going through the Small Larmor Radius Expansion, the general resonance condition is found.

(6.8) $$\omega - n\omega_c - k_{ll}v_{ll} = 0; n = 0, \pm1, \pm2, ...$$

iii. Cyclotron Absorption Mechanisms

For n=0, (in 80)
The absorption mechanisms in the plasma in this case are Transit Time Magnetic Pumping and Landau Damping. They correspond to parallel acceleration, and are important to current drive applications.
For n>0➔n=1,2,…
This corresponds to the resonances due to the left handed component of the field. For n=1, we have the fundamental cyclotron resonance $\omega = \omega_e + k_{ll}v_{ll}$. The wave accelerates the particles when it is in phase with their frequency. At higher harmonics (n=2,3,…), the wave accelerates the particles based on its propagation and being in phase with the particles.
For n<0➔n=-1,-2…
This corresponds to the resonances due to the right handed component of the field. This mechanism is significant only in high energy tails created by RF or NBI energy introduction.

iv. Scenarios

The most useful scenario today is the heating of a hydrogen minority in a D-(H) plasma. Other scenarios have been successfully used.

v. Database and Applications

Several high power ICH systems have been installed. A 22 MW system was coupled to the plasma in JET, and ICH systems have been injected into all sorts of plasmas such as L-mode, and ELMy H-mode. FW electron current drive has been tested in DIII-D and Tore-Supra showing good agreement with expected outcomes. Minority-ion current drive has been found allow control of the sawtooth frequency, and ICH systems have been used to produce a plasma in the presence of a static magnetic field.

An ICH system is designed for ITER coupling 50 MW through 3 ports.

b) Lower Hybrid Heating

Two waves coexist in the LH domain, the fast and slow waves. For them to uncouple and propagate inside the plasma, they must be from a launcher designed to launch waves with a parallel wavelength shorter than

(6.9) $$\lambda_{ll} = c/(N_c f)$$

Launchers designed to these specifications use an array of phased waveguides, or a grill.

The slow wave is launched into the plasma at a frequency above LH resonance and is efficiently absorbed by the plasma electrons. Because of an asymmetric N_{ll}, LH is used as a current drive method. The problem with LH heating is that the wave energy tends to propagate around the periphery of the plasma, and deposit its energy around the edge of the plasma, away from where it is needed.

As plasma density is increased, the LH wave goes from heating electrons to ions, then decay activity sets in, and no heating is accomplished. In large, hot plasmas, such as those planned for ITER, LH waves cannot usually reach the center.

c) Alfven Wave Heating

The compressional Alfven wave is launched into the plasma, and upon reaching the Alfven resonance, shear waves are produced which dissipate onto that magnetic surface. Alfven waves have been shown to produce little heating, but increase plasma density, and are therefore being studies for transport barrier purposes.

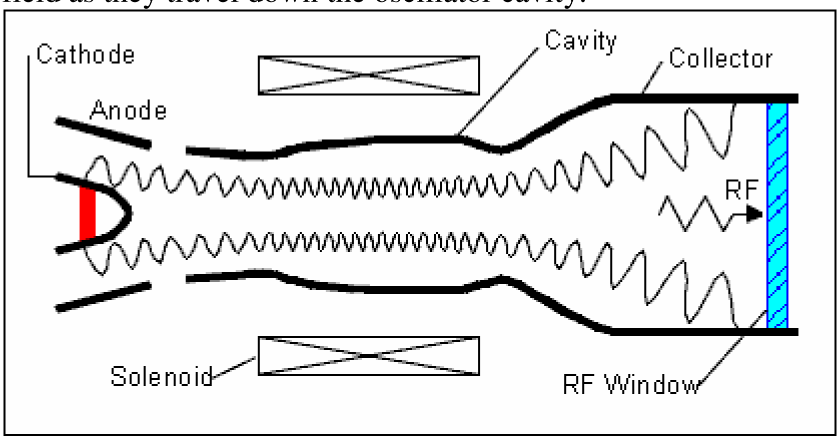
VII. Electron Cyclotron Wave Heating

Electron cyclotron waves heat a plasma at the fundamental electron cyclotron frequency, $\omega_{ce} = \frac{eB}{m_e}$ (about 100GHz to 200GHz) or a harmonic thereof, by heating plasma electrons which then in turn heat plasma ions due to collision heating. The ECRF wave is either an elliptically polarized wave at the cyclotron frequency for X-mode heating or a linearly polarized wave at the fundamental plasma frequency for O-mode heating. Due to the field intensity in the tokamak, the ECRF resonance surface is a thin plane in the radial direction extending vertically.

a) ECRF Generation

The requirement of high power high frequency RF sources has led to the use and improvement of existing RF sources to met the increasing power output demand.

High power ECRF is produced using a gyrotron RF source(Hoekzema Sect III.A). In a gyrotron, electrons spiral helically about the magnetic field produced in the oscillator cavity, and in the process azimuthally group into bunches, producing a high power RF field as they travel down the oscillator cavity.



b) ECRF Transport

Due to the losses present at high frequencies in dielectric material, either small coaxial cables(Stacey Sect. 12.4.2.1), or waveguides(Hoekzema Sect III.B) are used to inject the ECRF into the tokamak.

c) ECRF Launching

ECRF is accomplished through a grilled aperture structure. By adjusting the phase of the wave at each sub section of the grill, the array factor can be adjusted to launch the RF at any given angle into the plasma. Due to the high frequency, the ECRF wave is rarely reflected off of the vacuum-plasma interface(Hoekzema Sect 1).

d) ECRF Accessibility

Since the fundamental frequency EC wave reaches cutoff before it reaches resonance when propagation into an increasing magnetic field, the wave will be reflected unless it is injected from the high field side.

To allow low field side injection the second harmonic of the ECRF frequency is used, allowing the ECRF wave to reach the fundamental resonance surface before it reaches the second harmonic cutoff surface. The tradeoff is that gyrotrons for the second harmonic are not readily available and the absorption is weaker then that of the fundamental frequency.

This weaker absorption rate causes the wave heating to be distributed over a larger volume of the plasma and is not as controllable. For most modern tokamaks, first harmonic ECRF is used to heat the plasma in precise locations and is injected from the low field side.

VIII. Current Drive

Current Drive is important in a tokamak to drive the plasma current. It is preferable to use RF current drive because of its ability to be used in steady-state.

a) Current Drive Efficiency

Theoretically, a local efficiency of a current drive method can be defined as the ratio of driven current density J to power density deposited in plasma to create the current P_d. In an experiment, a scaled figure of merit $\eta_{CD} = \bar{n}_e R_0 I / P$. This quantity is proportional to $\bar{n}_e J / P_d$, allowing comparison of different machines and densities.

b) Current Drive by Pushing Electrons

The most direct manner of driving current with RF waves is injecting a wave with set frequency and **k** that is absorbed by passing electrons. Peaking the injected wave spectrum about a certain k_{ll} value causes asymmetry in the parallel velocity distribution causing current flow. This current drive becomes steady with a balance between the driving wave effect and collisional relaxation toward a Maxwellian distribution.

The choice of k_{ll} allows one to select $v_{ll,res}$ for resonance with either slow (~v_{te}) or fast electrons (several times v_{te}) that are passing the wave. Slow electrons are more easily pushed, but tend to relax collisionally towards a no-current state. Fast electrons are harder to push, but are not as collisional, providing longer time period current drive. It is desirable to aim for either very slow or very fast electrons, as this generates the most efficient current drive. However, at the lower limit, v_{ll}=0, only trapped electrons exist.

Driving fast electrons via Landau dampening with the Lower Hybrid wave has produced good results, with a fraction of the high-energy electrons downshifting in velocity and transferring their energy to slower electrons, enabling them to gain energy from the wave. Figure-of-merit values of 3-4x10^{19} have been seen in large tokamaks. However, penetration limits at higher densities and temperatures restrict its applicability in reactors.

The fast wave, and those waves to which it converts, have been used in driving current as they avoid the plasma penetration difficulty. Figures of merit from .45-.7x10^{19} have been realized through this method.

c) Current Drive by Asymetric Electron Collisionality

Circularly polarized electron-cyclotron waves can be used to drive current so that the $v_\perp$ of the electrons is increased, making their v-value greater than the normal electrons, and will therefore collide less and relax more slowly towards v_{ll}=0. This asymmetry creates a steady J_{ll} current density. It is advantageous to resonate with fast, noncollisional electrons.

The velocities and driven currents on each side of the resonant surface are opposite in sign, varying continuously with a R^{-1} dependence and zero on the resonant surface. This means that the asymmetry has to be significant for the currents not to cancel out. The polarization and launching location is determined with this in mind.

A maximum figure-of-merit of $.35 \times 10^{19}$ has been seen with this method. Localization of absorption on the electron cyclotron frequency surface makes this a candidate for tailoring tokamak current profiles.

d) Neutral Beam Injection Current Drive

Neutral beam injection involves shooting a beam of high energy neutrals into a plasma tangentially, which are then ionized by collisions, and subsequently lose energy in a directed manner, driving current. A figure-of-merit up to $.8 \times 10^{19}$ has been achieved in the JT-60 machine using negative ions in the beam generator to increase penetration.

e) Bootstrap Current

Consider a trapped electron and a fully passing electron in a plasma with r/R<<1. Through a kinetics analysis of conservation of angular momentum during collisions and constants of motion, we arrive at the equation

(8.1) $$eB_p\delta\langle r_i\rangle \approx -\Sigma\delta\langle p_i\rangle = \delta p_{llp}$$

This equation implies that the diffusion of the trapped electron out of the plasma be accompanied by net negative parallel kinetic momentum flux to passing electrons, hence a positive current.

Ambipolar diffusion outward of particles from the center of the toroidal plasma column contributes, through radial particle flux and the magnetic field interaction, a toroidal bootstrap current density.

Bootstrap current is very important for the future of fusion power and tokamaks. It provides for a high plasma current providing desirable plasma parameters while eliminating the need for direct current drive through RF or NBI systems. Many tokamaks, both present day and near-term, use or plan to utilize a high bootstrap current. Research is being performed on designing a fully boostrap-driven plasma current in a tokamak

Several combined-wave current drive systems have been proposed including Fast electron pushing + EC resonance, Fast wave + LH, and NBI + IC resonance.

Problems with CD waves include

LH: penetration
+ NBI: penetration, low figure-of-merit
- NBI: low figure-of-merit
FW-high frequency: penetration
Others: low figure-of-merit, generation difficulties

IX. Selected RF Heating and Current Drive in Use Today

	system	frequency (MHz)	maximum power coupled to plasma (MW)	number of antennas (x #of straps)
Fast Wave	Asdex-Upgrade	30-120	5.7	4(x2)
	C-mod	80.0	3.5	2(x2)
	DIII-D	30-120	3.6	3(x4)
	HT-6M	14-45	0.6	1(x1)
	JET	23-57	22.0	4(x2)
	JT-60U	102-131	7.0	2(x2)
	TEXTOR	25-38	3.6	2(x2)
	TFTR	30-76	11.4	4(x2)
	Tore-Supra	35-80	9.5	3(x2)
	ITER***	40-75	50.0	-

Lower Hybrid	system	frequency (GHz)	maximum power coupled to plasma (MW)	waveguides	Current Drive Efficiency (x10^20 A/(Wm^2))	
	COMPASS	1.3	0.6	8.0		
	FT-U	8.0	5.5	72 (6 ant.)	0.2, 0.5*	
	HT-6M	2.5	0.1	8.0		
	JET	3.7	5.0	384.0	0.35, 0.45**	
	JT-60U	1.74-2.23	10.0	24.0		0.270
		1.74-2.23	2.0	48.0		
	PBX-M	4.6	1.3	32.0		
	TdeV	3.7	8.0	32.0		
	Tore-Supra	3.7	0.1	256 (2 ant.)		
	TRIAM-1M	2.5		4.0		
	ITER***	5.0	50.0		0.45-.55	

* with full current drive
** with LH & ICRH hybrid
*** expected date 2016

X. References

[1] Stacey, Weston M. Fusion Plasma Physics. 1st ed. Weinheim: Wiley-VCH, 2005.

[2] Chen, Francis F. Introduction to Plasma Physics and Controlled Fusion, Volume 1: Plasma Physics. 2nd ed. Plenum Press 1984

[3] ITER Technical Basis, Chap. 2.5 Additional Heating and Current Drive, G A0 FDR 101-07-13 R1.0 (2001).

[4] Koch, R. "The Coupling of Electromagnetic Power to Plasmas." Transactions of Fusion Technology os 37 (2000): 145-154.

[5] Hutchinson, I. H. "Electromagnetic Waves in Plasmas." Introduction to Plasma Physics. Dept. of Phys., MIT. Fall 2006 <http://silas.psfc.mit.edu/introplasma/chap5.html>.

[6] Fitzpatrick, Richard. "Waves in Cold Plasmas." Introduction to Plasma Physics. Dept. of Phys., U. of Texas at Austin. Fall 2006 <http://farside.ph.utexas.edu/teaching/plasma/lectures/node41.html>.

[7] Koch, R. "The Ion Cyclotron, Lower Hybrid and Alfeven Wave Heating Methods." Transactions of Fusion Technology os 37 (2000): 155-162.

[8] Hoekzema, J. A. "Electron Cyclotron Waves." Transactions of Fusion Technology os 37 (2000): 163-169.