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Sensitivity of the interpretation of the experimental ion thermal diffusivity to the determination of the ion conductive heat flux

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A moments equation formalism for the interpretation of the experimental ion thermal diffusivity from experimental data is used to determine the radial ion thermal conduction flux that must be used to interpret the measured data. It is shown that the total ion energy flux must be corrected for thermal and rotational energy convection, for the work done by the flowing plasma against the pressure and viscosity, and for ion orbit loss of particles and energy, and expressions are presented for these corrections. Each of these factors is shown to have a significant effect on the interpreted ion thermal diffusivity in a representative DIII-D [J. Luxon, Nucl. Fusion **42**, 614 (2002)] discharge. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4873385]

I. INTRODUCTION

Interpretation of the measured radial profiles of density and temperature in the plasma edge in terms of the underlying particle and energy transport mechanisms has long been and remains (e.g., Refs. 1–9) an active area of research in tokamak plasma physics, in large part because of the perceived importance of the edge temperature in determining the core temperature in future tokamaks (e.g., Ref. 10). In many codes used for the prediction of plasma density and temperature profiles, particle and energy transport are assumed to satisfy Fick's ($\Gamma = -D\,\partial n/\partial r$) and Fourier's ($q = -n\chi\partial T/\partial r$) Laws for the diffusive particle and conductive energy fluxes, respectively. However, these Laws have sometimes been found to lead to unphysical values being required for the particle D and/or thermal χ diffusivities in order to match measured particle and temperature profiles.[8] A rigorous derivation of the particle flux from the first two moments of the Boltzmann equation (particle and momentum balance)[11,12] showed that the radial particle flux must satisfy a pinch-diffusion relation $\left(\Gamma = -D\partial(nT)/\partial r + nV_{pinch}\right)$, where the pinch term involves electromagnetic and neutral beam forces, and the particle diffusion coefficient is specified in terms of the momentum exchange frequencies. One purpose of this paper is to explore the possibility of a similar relation for the ion thermal conductive flux q. More generally, the broad purpose of this paper is to explore some further theory-based extensions of the methodology used to interpret thermal transport in the plasma edge.

II. MOMENTS EQUATIONS

Writing the Boltzmann equation as $Bf_j = C\left(f_j, \sum_{k\neq j} f_k\right) + S_j$, where B is the Boltzmann operator involving differentials in time, space, and velocity, C is the collision operator, $f_j(r, V_j)$ is the distribution function of species j, and S_j is the source rate of species j, the moments equations are obtained by multiplying by $Z_n(V_j)$ and integrating over V_j. The first four weighting factors used in this paper are $Z_0 = 1$, $Z_1 = \mathbf{V}_j$, $Z_2 = \frac{1}{2}m(\mathbf{V}_j \cdot \mathbf{V}_j)$, and $Z_3 = \frac{1}{2}m(\mathbf{V}_j \cdot \mathbf{V}_j)\mathbf{V}_j$. Choosing a normalization such that $n_j = \int f_j(\mathbf{V}_j)d\mathbf{V}_j$, the average velocity $\bar{\mathbf{V}}_j \equiv \int \mathbf{V}_j f_j(\mathbf{V}_j)d\mathbf{V}_j/n_j$, etc., and decomposing the velocity variable vector into the average velocity plus a random velocity $\mathbf{V}_j = \bar{\mathbf{V}}_j + \mathbf{w}_j$ leads to[13]

$$
\begin{aligned}
\nabla\cdot\left(n_j\bar{\mathbf{V}}_j\right) &= -\frac{\partial n_j}{\partial t} + S_j^0,\\
n_jm_j\left(\bar{\mathbf{V}}_j\cdot\nabla\right)\bar{\mathbf{V}}_j + \nabla p_j + \nabla\cdot\Pi_{2j} &= -n_jm_j\frac{\partial\bar{\mathbf{V}}_j}{\partial t} + n_je_j\left(\mathbf{E} + \bar{\mathbf{V}}_j\times\mathbf{B}\right) + \mathbf{R}_j^1 + \left(\mathbf{S}_j^1 - m_j\bar{\mathbf{V}}_jS_j^0\right),\\
\nabla\cdot\bar{\mathbf{Q}}_j &= -\frac{3}{2}\frac{\partial p_j}{\partial t} + n_je_j\bar{\mathbf{V}}_j\cdot\mathbf{E} + R_j^2 + S_j^2,
\end{aligned}
\tag{1}
$$

where the superscripts on R and S indicate the Z_n moment of the collision and source terms, respectively, and the "2" subscript on Π indicates a viscosity tensor of second order.

The quantity $\bar{\mathbf{Q}}_j \equiv \frac{1}{2}m_j\int f_j(V_j)(\mathbf{V}_j\cdot\mathbf{V}_j)\mathbf{V}_jd\mathbf{V}_j$ is the total energy flux for species j, which may be shown[13] to consist of thermal and kinetic energy plus the work done by the outflowing plasma against pressure and viscous shear. It can be represented in terms of the lower order velocity moments (density, average velocity), the second order velocity moment (pressure, or equivalently temperature T = p/n), and a new third order velocity moment identified as the thermal conduction, $\mathbf{q}_j \equiv \frac{1}{2}m_j\int(\mathbf{w}_j\cdot\mathbf{w}_j)\mathbf{w}_jd\mathbf{w}_j$

$$\bar{\mathbf{Q}}_j \equiv \left(\frac{1}{2}n_j m_j(\bar{\mathbf{V}}_j\cdot\bar{\mathbf{V}}_j)\bar{\mathbf{V}}_j + \frac{3}{2}p_j\bar{\mathbf{V}}_j + \{p_j\bar{\mathbf{V}}_j + \bar{\mathbf{V}}_j\cdot\Pi_{2j}\} + \mathbf{q}_j\right). \tag{2}$$

The first term represents the convection of rotational kinetic energy, the second term represents the convection of thermal plasma energy, the third term represents the work done by the flowing plasma against the pressure and viscous stress, and the last term represents the conduction of thermal plasma energy.

A moments derivation of this sort always has the lack of closure problem–no matter at which order the set of moments equations are terminated there is at least one more unknown than equation. For example, the first of Eqs. (1) has two unknowns, the density (zeroth order velocity moment) and the average velocity (first order velocity moment); the set of the first two equations contains a third unknown, the pressure (second order velocity moment); and the set of the first three equations has a fourth unknown, the thermal conduction (third order velocity moment).

The fourth moment equation (Eq. 5.9 or 5.29 of Ref. 13) involves the thermal conduction flux $\mathbf{q}_j$ and both higher and lower order moment unknowns. Inclusion of this fourth moment equation, which involves electromagnetic "energy pinch" terms, in the set of Eq. (1) presumably would provide a more accurate set of equations and extend the issue of closure to one higher order. However, inclusion of this fourth moment equation results in an extremely formidable set of moments equations which would not likely be much used in either the interpretation or prediction of transport in the edge plasma. Thus, while it seems that there is a "pinch-diffusion" relation for the conductive heat flux, it seems more practical to use Eqs. (1) and (2) together with a transport theory closure relation.

III. RADIAL THERMAL CONDUCTIVE ENERGY FLUX

Once the total radial heat and particle fluxes, density, pressure, and average velocities are determined from experimental measurement and by solving Eq. (1), the conductive thermal flux can be determined from the radial component of Eq. (2)

$$q_{rj} = \bar{Q}_{rj} - \frac{3}{2}p_j\bar{V}_{rj} - \frac{1}{2}n_j m_j(\bar{\mathbf{V}}_j\cdot\bar{\mathbf{V}}_j)\bar{V}_{rj} - (p_j\bar{V}_{rj} + \bar{\mathbf{V}}_j\cdot\Pi_{2j}\cdot\hat{\mathbf{n}}_r), \tag{3}$$

which requires in addition the evaluation of the viscous term.

Using Braginskii's decomposition of the viscous stress tensor[14,15] generalized to toroidal flux surface geometry,[16,17] the viscous term becomes

$$\begin{aligned}\bar{\mathbf{V}}_j\cdot\Pi_{2j}\cdot\hat{\mathbf{n}}_r = \eta_o\{&-1/3(\partial\bar{V}_{\theta j}/\partial\ell_\theta) + (\bar{V}_{\theta j}/RB_\theta)[\partial(RB_\theta)/\partial\ell_\theta] \\ &+ f_p R[\partial(V_{\phi j}/R)/\partial\ell_\theta]\},\end{aligned} \tag{4}$$

where $f_p \equiv B_\theta/B_\phi$ and the Shaing-Sigmar extension[18] of the parallel viscosity coefficient to arbitrary collisionality is $\eta_o = n_j m_j V_{thj} qRf_j$, with $f_j \equiv \nu^*_{jj}/(1+\varepsilon^{1.5}\nu^*_{jj})(1+\nu^*_{jj}), \varepsilon \equiv r/R$, $\nu^*_{jj} \equiv \nu_{jj} qR/V_{thj}$, and here q is the safety factor. For the circular toroidal flux surface representation $R = R_o(1+\varepsilon\cos\theta)$, $B = B_o/(1+\varepsilon\cos\theta)$, $RB_\theta \neq f(\theta)$ and the thermal conduction flux can be calculated from

$$\begin{aligned} q_{rj} = Q_{rj} - T_j n_j \bar{V}_{rj}\Bigg\{&\frac{3}{2} + \left(\frac{\bar{V}_j}{V_{thj}}\right)^2 \\ &+ \left[1 + 2qRf_j\left(f_p\frac{\bar{V}_{\phi j}}{V_{thj}}\delta_\phi - \frac{1}{3}\frac{\bar{V}_{\theta j}}{V_{thj}}\delta_\theta\right)\right]\Bigg\}.\end{aligned} \tag{5}$$

The first term in the {} represents thermal convection, and the second term represents the convection of rotation energy. The last term represents the work done against the pressure and the viscous stress and is driven by poloidal flow asymmetries $\delta_\theta \equiv (r/\bar{V}_\theta)(\partial\bar{V}_\theta/\partial\ell_\theta)$, $\delta_\phi \equiv (r/(\bar{V}_\phi/R))(\partial((\bar{V}_\phi/R)/\partial\ell_\theta)$, the calculation of which is discussed in Refs. 19–21.

IV. TRANSPORT CLOSURE RELATIONS

The more or less standard choice for the transport closure relation is the Fourier heat conduction Law

$$\mathbf{q}_j = -n_j\chi_j\nabla T_j. \tag{6}$$

Note that the use of Eq. (6) implicitly assumes a thermal diffusive mechanism that produces a thermal energy flux proportional to the density and to the temperature gradient. This relation is satisfied by neoclassical theory[15] (although there is also a density gradient term in some cases) and is used to represent heat conduction caused by various small-scale instabilities,[22] but is not consistent in form with all heat conduction theories (e.g., the paleoclassical theory[9]). For interpretation of experimental transport coefficients for the purpose of comparison with transport theories having a form different from Eq. (6), that different form should be adopted as the closure relation at this point.

Use of the Fourier Law for heat conduction, as is common practice,[8] allows the radial thermal diffusivity to be inferred from the measured temperature and density distributions, $\left(\chi_j^{exp} = -q_{rj}/n_j^{exp}\nabla T_j^{exp} \equiv q_{rj}L_{T_j}^{exp}/n_j^{exp}T_j^{exp}\right)$. If another form of the thermal conduction relation was used, it should still be possible to infer the thermal transport coefficient from the experimental data in a similar manner.

V. INTERPRETATION OF TRANSPORT COEFFICIENTS

As mentioned previously and discussed in Refs. 11, 12, and 23, and elsewhere, the second of Eq. (1) requires that the radial particle flux must satisfy a pinch-diffusion relation $(\Gamma = -D\partial(nT)/\partial r + nV_{pinch})$, where the pinch term incorporates the electromagnetic and external force terms, and the particle diffusion coefficient is specified in terms of the momentum exchange frequencies arising from the viscous, inertial, collisional, and source terms. The magnitude of the radial particle flux is determined by the first of Eq. (1). The magnitude of the radial energy flux is determined by solving the third of Eq. (1), and the conductive radial energy flux is then determined from Eq. (3) or (5).

For ions in the tokamak plasma edge, there is a further factor that affects particle and energy fluxes being "transported" radially outward. Ions with sufficient energy can access orbits that will carry them quickly outward across the last closed flux surface without further collisions or other interactions with the plasma ions and electrons. Since these "ion orbit loss" particles do not interact with the plasma ions and electrons being carried outward by other transport processes, they should not be included in the total number of particles or energy for the purpose of interpreting these other transport processes in the plasma. The ion orbit loss of the thermalized plasma ions and energy can be calculated from considerations of conservation of energy, canonical angular momentum, and magnetic moment.[24–28] The thermal ion orbit losses (and the similarly calculated beam ion orbit losses) can be incorporated directly into the solution of the particle continuity and energy balance equations for the radial ion and energy fluxes from the first and third of Eq. (1), which are written (in the slab approximation) for a single ion species and the electrons as[28]

$$\begin{aligned}
\frac{\partial \hat{\Gamma}_{rj}}{\partial r} &= -\frac{\partial n_j}{\partial t} + N_{nbi}\left(1-2f_{nbi}^{iol}\right) + n_e n_o \langle\sigma\upsilon\rangle_{ion} - 2\frac{\partial F_{orbj}}{\partial r}\hat{\Gamma}_{rj},\\
\frac{\partial \hat{Q}_{rj}}{\partial r} &= -\frac{\partial}{\partial t}\left(\frac{3}{2}n_j T_j\right) + q_{nbi}^{j} f_{nbi}^{iol} - q_{j\to e} - n_j n_o^c \langle\sigma\upsilon\rangle_{cx}\frac{3}{2}\left(T_j - T_o^c\right)\\
&\quad - \frac{\partial E_{orbj}}{\partial r}\hat{Q}_{rj},\\
\frac{\partial \hat{Q}_{re}}{\partial r} &= -\frac{\partial}{\partial t}\left(\frac{3}{2}n_e T_e\right) + q_{nbi}^{e} f_{nbi}^{iol} + q_{j\to e} - n_e n_z L_z(T_e),
\end{aligned} \tag{7}$$

where N_{nbi} and $q_{nbi}^{j,e}$ are the neutral beam particle and energy sources, the $\langle\sigma\upsilon\rangle$ are ionization and charge exchange rates, n_o and n_z are the neutral and impurity densities, $q_{j\to e}$ is the ion to electron energy exchange rate, L_z is the impurity radiation emissivity, T_o^c is the temperature of cold recycling neutrals, $\Gamma_{rj} \equiv n_j \bar{V}_{rj}$ and the carat indicates that the radial particle and heat fluxes are calculated including the effects of ion orbit loss. A formalism based on the conservation of canonical angular momentum, energy, and magnetic moment for the calculation of cumulative (in radius) thermal ion particle and energy loss fractions F_{orbj} and E_{orbj} is described in Refs. 24–28, and the differential ion orbit loss rates from the ion radial particle and energy fluxes are $\left(\partial F_{orbj}/\partial r\right)\hat{\Gamma}_{rj}$. and $\left(\partial E_{orbj}/\partial r\right)\hat{Q}_{rj}$. The (local) fast beam ion loss fraction f_{nbi}^{iol} can be calculated by the same procedure and is included for completeness, although beam ion deposition in the edge pedestal is quite small. The factor of 2 in the first of Eq. (7) takes into account the inward main ion return current from the SOL, which is necessary to maintain charge neutrality in the presence of the beam and thermal plasma ion orbit losses.[27]

Using these radial particle and heat fluxes evaluated with the experimental density and temperature, the experimental thermal diffusivities interpreted from the measured data can be calculated from the heat conduction relation of Eq. (6)

$$\chi_{j,e}^{\exp} = \frac{\hat{q}_{j,e}}{-n_{j,e}^{\exp}\left(\frac{\partial T_{j,e}^{\exp}}{\partial r}\right)} = \frac{\left(\hat{Q}_{rj,e} - 0.5\{\}_{j,e}\hat{\Gamma}_{rj,e}\right)}{-n_{j,e}^{\exp}\left(\frac{\partial T_{j,e}^{\exp}}{\partial r}\right)}, \tag{8}$$

where the $\{\}_{j,e}$ term for ions is given in Eq. (3) or (5) and for electrons is $5\,T_e^{\exp}$.

VI. APPLICATION TO A DIII-D H-MODE DISCHARGE

Data from DIII-D[29] ELM-free, H-mode (Edge-Localized Mode free, High confinement mode) discharge 118 897[30,31] at 2140 ms were used to evaluate the above formalism in order to assess the sensitivity of inferred ion thermal conduction flux and ion thermal diffusivities to taking into account the above corrections for (i) convection of thermal energy, (ii) convection of rotational energy, (iii) work done by the flowing plasma against pressure and viscosity, and (iv) ion orbit loss of particles and energy. The ion temperature varied from about 650 eV at rho = 0.86 to about 50 eV at the separatrix, and the electron density varied from 0.75×10^{19} to $0.7 \times 10^{18}/\mathrm{m}^3\mathrm{s}$ over this range. Further details of the data may be found in Ref. 31. The measured toroidal and poloidal rotation velocities (for the ≈4% carbon impurity) were in the range of 1–10 km/s in this shot with relatively low neutral beam injection power. Deuterium toroidal velocities were estimated from perturbation theory[27,30] and deuterium poloidal velocities were inferred from radial momentum balance.[31]

The ion conductive thermal fluxes determined by using the experimental data to evaluate Eqs. (1) and (3) are displayed in Fig. 1. The qi0 is the total energy flux calculated from the third of Eq. (1). When this flux is corrected for the convection of ion thermal energy, the resulting conductive flux qi1 is reduced substantially, but the subsequent reduction for the convection of rotational energy does not have a large further effect on qi2. Further corrections for the work done by the flowing plasma against the pressure and viscous stress again cause a substantial reduction in the resulting conductive flux qi3. (The largest part of which is from the pressure term even though the viscous term was evaluated

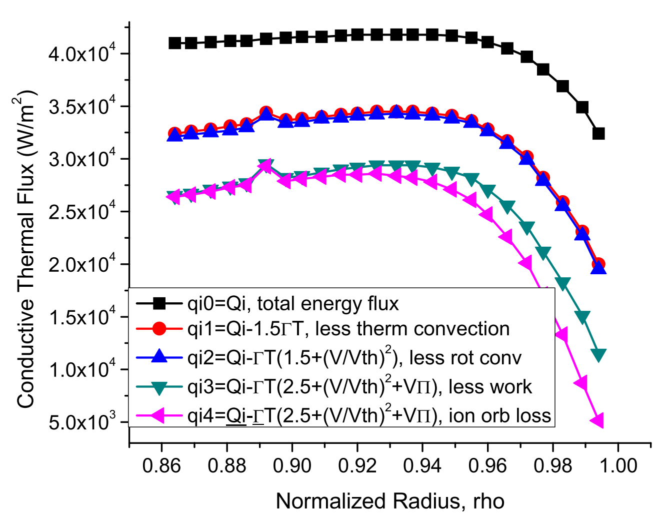

FIG. 1. Ion thermal conduction flux.

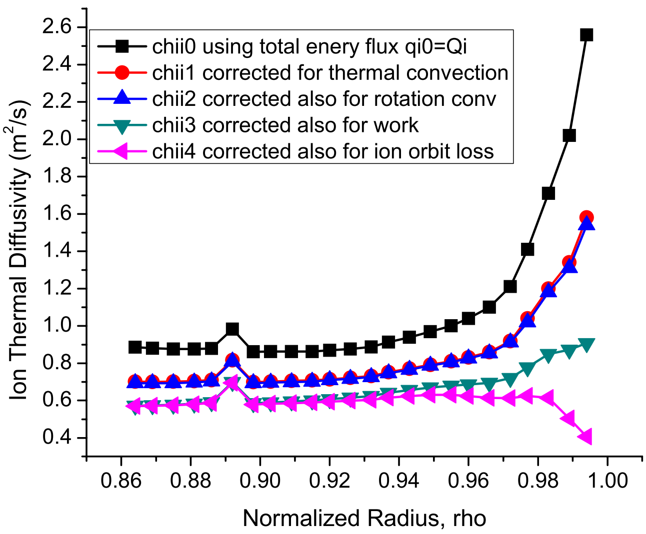

FIG. 2. Ion thermal conduction flux

assuming a relatively large $\delta_\phi \approx \delta_\theta \approx 10\%$ poloidal asymmetry.) Finally, taking into account ion orbit loss reductions to the total ion particle (Γ) and energy (Q) fluxes produces a further significant reduction in the edge pedestal to a total conductive ion heat flux of qi4.

Using the different values of the ion conductive heat flux, along with the measured temperatures and density, in Fig. 1 in Eq. (8) to interpret the ion thermal diffusivity, results in the values displayed in Fig. 2. The chii0 results from using the total ion energy flux qi0, the chii1 from using the qi1 flux corrected for convection of thermal energy, etc. Clearly, correcting the total energy flux calculated by using the experimental data to evaluate Qi from the third of Eq. (1) to account for thermal energy convection, work done by the flowing plasma against the pressure and ion orbit loss all make a substantial difference on the interpreted ion thermal diffusivity. In this discharge correcting for the convective transfer of rotational energy is unimportant. (We note that the rotation energy correction may be more important in the core plasma, where the rotation toroidal velocities are larger.) However, the rotation and viscous corrections would be larger in H-mode discharges with larger directed neutral beam injection in which experimental rotation velocities an order of magnitude larger than those in this discharge are found in the edge plasma.[27] (The "bump" in the curves at rho = 0.89 in Figs. 1 and 2 is an artifact of interfacing different computational algorithms.)

VII. SUMMARY AND CONCLUSIONS

The inferred experimental ion thermal diffusivity is shown to be sensitive to the determination of ion conductive heat flux. The radial ion total energy flux determined from solving the ion energy balance equation must be corrected for the convection of thermal and rotation energy, for the work done by the flowing plasma against the pressure and viscous stress, and for ion orbit loss. Each of these factors is shown to have a significant effect in a representative DIII-D H-mode discharge.

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