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Inclusion of ion orbit loss and intrinsic rotation in plasma fluid rotation theory

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The preferential ion orbit loss of counter-current directed ions leaves a predominantly co-current ion distribution in the thermalized ions flowing outward through the plasma edge of tokamak plasmas, constituting a co-current intrinsic rotation. A methodology for representing this essentially kinetic phenomenon in plasma fluid theory is described and combined with a previously developed methodology of treating ion orbit particle and energy losses in fluid theory to provide a complete treatment of ion orbit loss in plasma fluid rotation theory. © *2016 AIP Publishing LLC*.
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I. INTRODUCTION

A methodology previously has been developed for calculation of the ion-orbit-loss of particles, momentum, and energy in the thermalized outflowing ion distribution in tokamak edge plasmas.[1,2] The calculation is based on combining the requirements for the conservation of canonical angular momentum, energy, and magnetic moment to obtain an equation for the minimum speed a particle at a given location (r, θ) on an internal flux surface with a given pitch angle ζ_0 (with respect to $\mathbf{B}$) must have in order to reach a given location on the last closed flux surface (LCFS), $V_{0\min}(\zeta_0)$. This minimum $V_{0\min}(\zeta_0)$ enables the calculation of the fractions of particles, ion energy, and ion momentum flowing across an internal flux surface in the edge plasma that consists of free-streaming ion-orbit-loss particles.[1,2] This methodology and similarly based methodologies have been applied to explain the intrinsic co-current rotation peaking of toroidal rotation (which is an experimental signal of ion orbit loss (IOL)) measured in the edge of several DIII-D discharges[3–7] and in an EAST discharge.[8,9]

IOL is essentially a "kinetic" effect very much associated with the directionality of the ion distribution, so the question of how to incorporate such a kinetic effect into the "fluid" theory commonly used for edge plasma analyses naturally arises. It has been previously shown[10] that the particle and energy ion orbit losses can be incorporated into the radial continuity and energy balance equations as differential loss rates proportional to the local particle and energy fluxes at each radius, and that the continuity equation can be further corrected to account for the return current necessary to maintain charge neutrality. However, inclusion of the toroidal and poloidal ion orbit momentum gain/loss into the toroidal and poloidal momentum balance equations at each radial location is less straightforward and is the main topic of this paper.

II. ION ORBIT LOSS OF PARTICLES AND ENERGY

Plasma fluid theory treats an outwardly flowing distribution of plasma ions that are at least implicitly assumed to be near Maxwellian. As these ions near the edge, some of the higher energy ions in the distribution become able to access orbits that carry them outside the plasma. The methodology employed in Refs. 1, 2, and 10 describes the calculation of radially cumulative particle and energy ion orbit loss fractions of the outwardly flowing ion distribution, in effect assuming that the ion distributions remain Maxwellian with a temperature that decreases with radius, as in fluid theory, but with the distribution being "chopped off" above a certain minimum energy which depends on the directionality of the ion and the parameters of the edge plasma, and which decreases with radius.

The cumulative fractions of the outflowing thermalized ion and ion energy fluxes that are in the form of ion orbit loss particles streaming outward across an internal flux surface are the fractions of these distributions associated with particles with $V_0(\rho, \zeta_0) > V_{0\min}(\rho, \zeta_0)$

$$F_{orb} \equiv \frac{n_{loss}}{n_{tot}} = \frac{\int_{-1}^{1}\left[\int_{V_{0\min(\zeta_0)}}^{\infty} R_{loss}^{iol}(V_0,\zeta_0) f(V_0) V_0^2 dV_0\right] d\zeta_0}{\int_{-1}^{1}\left[\int_0^{\infty} f(V_0) V_0^2 dV_0\right] d\zeta_0}$$
$$= \frac{1}{2}\int_{-1}^{1} \langle R_{loss}^{iol}(V_{0\min(\zeta_0)},\zeta_0)\rangle_n \left[\frac{\Gamma(3/2, \varepsilon_{\min(\zeta_0)})}{\Gamma(3/2)}\right] d\zeta_0,$$
$$E_{orb} \equiv \frac{E_{loss}}{E_{tot}}$$
$$= \frac{\int_{-1}^{1}\left[\int_{V_{0\min(\zeta_0)}}^{\infty} R_{loss}^{iol}(V_0,\zeta_0)\left(\frac{1}{2}mV_0^2\right) f(V_0) V_0^2 dV_0\right] d\zeta_0}{\int_{-1}^{1}\left[\int_0^{\infty}\left(\frac{1}{2}mV_0^2\right) f(V_0) V_0^2 dV_0\right] d\zeta_0}$$
$$= \frac{1}{2}\int_{-1}^{1} \langle R_{loss}^{iol}(V_{0\min(\zeta_0)},\zeta_0)\rangle_E \left[\frac{\Gamma(5/2, \varepsilon_{\min(\zeta_0)})}{\Gamma(5/2)}\right] d\zeta_0, \quad (1)$$

where R_{loss}^{iol} is the fraction of such particles leaving the confined plasma that do not return, $\varepsilon_{\min} \equiv mV_{0\min}^2(\zeta_0)/2T$, Γ is the gamma function, and a Maxwellian distribution has been used to evaluate the integrals over ion speed. $V_{0\min}(\zeta_0)$ is the minimum speed of a particle at a given location on an internal flux surface with pitch angle ζ_0 (with respect to $\mathbf{B}$) which can reach a given location on the LCFS, and can be determined[1,2] from conservation of canonical angular momentum, energy, and magnetic moment, which yield an equation for this minimum value

$$V_0^2\left[\left(\frac{R_0 f_{\varphi 0}}{R\ f_\varphi}\zeta_0\right)^2 - 1 + \left(1-\zeta_0^2\right)\left|\frac{B}{B_0}\right|\right] + V_0\left[\frac{2e(\psi_o-\psi)}{Rmf_\varphi}\left(\frac{R_0 f_{\varphi 0}}{R\ f_\varphi}\zeta_0\right)\right] + \left[\left(\frac{e(\psi_0-\psi)}{Rmf_\varphi}\right)^2 - \frac{2e(\phi_0-\phi)}{m}\right] = 0, \quad (2)$$

where ϕ is the electrostatic potential and the quantity $\zeta_0 = V_{\|0}/V_0$ is the cosine of the initial guiding center velocity relative to the toroidal magnetic field direction. Equation (2) is quite general with respect to the flux surface geometry representation of R, B, and the flux surfaces ψ. Equations (1) and (2) display the important dependence of the ion orbit loss on electric field (gradient of ϕ) first noted in Ref. 11.

The $F_{orb}(\rho,\zeta_0,\theta_0,\theta_s)$ and $E_{orb}(\rho,\zeta_0,\theta_0,\theta_s)$ are calculated at a number of locations θ_0 on the internal flux surface and θ_s on the LCFS, and are appropriately averaged[1,2] to obtain a value characterizing the internal flux surface ρ. These factors are incorporated into the fluid continuity and energy balance equations (in the slab approximation) as differential losses proportional to the particle and energy fluxes[10]

$$\frac{\partial\hat{\Gamma}_{ri}}{dr} = -\frac{\partial n_i}{\partial t} + N_{nbi}\left(1-2f_{nbi}^{iol}\right) + n_e\nu_{ion} - 2\frac{\partial F_{orbi}}{\partial r}\hat{\Gamma}_{ri},$$
$$\frac{\partial\hat{Q}_{ri}}{dr} = -\frac{\partial}{\partial t}\left(\frac{3}{2}n_iT_i\right) + q_{nbi}^i\left(1-\alpha e_{nbi}^{iol}\right) - q_{ie} - n_in_o^c\langle\sigma v\rangle_{cx}\frac{3}{2}\left(T_i-T_o^c\right) - \frac{\partial E_{orbi}}{\partial r}\hat{Q}_{ri}, \quad (3)$$

where the carat indicates that the radial particle and heat fluxes are calculated including the effects of ion orbit loss (and return current) represented by (cumulative in radius) thermalized ion particle and energy loss fractions F_{orbi} and E_{orbi} and by the (local) fast beam ion loss fraction f_{nbi}^{iol} and beam ion energy loss fraction e_{nbi}^{iol} calculated similarly to F_{orbi} and E_{orbi} except for the beam ions being mono-directional and mono-energetic. For co-current injection, nearly all the beam energy is deposited collisionally in the plasma[12] ($\alpha^{co}=0$). For counter-current injection, not all the beam energy is absorbed in the plasma, even though the jXB energy deposition due to radial movement of the lost beam ions heats the plasma, and calculations for MAST[12] indicate ($0.5<\alpha^{ctr}<1$). The quantities N_{nbi} and q_{nbi} are the source rates of neutral beam particles and energy in the plasma. The factor of 2 in the first of Eqs. (3) arises from taking into account the divergence of the inward main ion currents from the SOL which are necessary to maintain charge neutrality in the presence of the beam and thermalized plasma ion orbit losses.[13]

Making use of the method of integrating factors, it can be shown that Eqs. (3) have solutions of the form

$$\hat{\Gamma}_{ri}(r) = \hat{\Gamma}_{ri}(r_0) + \int_{r_0}^{r} S_{ni}(r')e^{-2[F_{orbi}(r)-F_{orbi}(r')]}dr',$$
$$\hat{Q}_{ri}(r) = \hat{Q}_{ri}(r_0) + \int_{r_0}^{r} S_{Ei}(r')e^{-[E_{orbi}(r)-E_{orbi}(r')]}dr', \quad (4)$$

where r_0 is the innermost radius of the calculation (inside of which the ion orbit loss of thermalized ions is unimportant) and S_{ni} and S_{Ei} are the net sources (sources minus sinks) of ions and ion energy, respectively (i.e., all the terms except the last on the right sides of Eqs. (3)). Thus, ion orbit loss exponentially attenuates the thermalized ion particle and energy fluxes that would be calculated in their absence.

III. INTRINSIC ROTATION DUE TO ION ORBIT LOSS

We now extend this formalism to also take into account momentum ion orbit loss, which is related to intrinsic rotation. The preferential ion orbit loss of counter-current ions causes an effective co-current intrinsic rotation of the residual ions in the edge plasma due to the preferential retention of co-current direction ions. The net co-current parallel (to the plasma current) rotation velocity at any flux surface is determined by the net counter-current directed fraction of ion orbit momentum loss that has taken place inside of that flux surface[1]

$$\Delta V_\|(\rho) = 2\pi\int_{-1}^{1}d\zeta_0\left[\int_{V_{\min}(\zeta_0)}^{\infty} R_{loss}^{iol}(V_0\zeta_0)V_0^2f(V_0)dV_0\right]_\rho = 4\pi M_{orb}(\rho)\left[\int_0^\infty (V_0)V_0^2f(V_0)dV_0\right]_\rho, = \frac{2\Gamma(2)}{\pi^{1/2}}M_{orb}(\rho)V_{th}(\rho) = \frac{2}{\pi^{1/2}}M_{orb}(\rho)\sqrt{\frac{2kT_{ion}(\rho)}{m}}, \quad (5)$$

where

$$M_{orb}(\rho) \equiv \frac{M_{loss}}{M_{tot}} = \frac{R_{loss}^{iol}\int_{-1}^{1}\left[\int_{V_{0\min}(\zeta_0)}^{\infty}(mV_0\zeta_0)V_0^2f(V_0)dV_0\right]d\zeta_0}{2\int_0^\infty (mV_0)V_0^2f(V_0)dV_0} = \frac{R_{loss}^{iol}\int_{-1}^{1}\zeta_0\Gamma(2,\varepsilon_{\min}(\rho,\zeta_0))d\zeta_0}{2\Gamma(2)} \quad (6)$$

is the net (co-current minus counter-current) fraction of the outflowing thermalized ion parallel momentum that has been lost from the plasma by ions accessing orbits that cross the separatrix. Different values of M_{orb} are calculated for the different thermalized ion species (e.g., deuterium and carbon), resulting in different parallel intrinsic velocities, which can be projected into toroidal and poloidal intrinsic rotation velocities for the different ion species.

This prediction of intrinsic co-current rotation by ion orbit loss has been confirmed by measurements of co-current peaking of the toroidal velocity just inside the LCFS in DIII-D[5–7] and EAST.[8]

IV. EFFECTIVE FLUID TOROIDAL MOMENTUM SOURCE FROM INTRINSIC ROTATION

The "kinetic" intrinsic rotation calculated in Sec. III can just be added to the fluid rotation calculated from momentum balance, as discussed in the Secs. V and VI. However, it may be convenient in some applications to instead define an effective IOL fluid momentum that would produce this same rotation if included in the fluid momentum balance equations. For a two-species plasma, the fluid toroidal momentum balance equations can be written[14]

$$
\begin{aligned}
n_j m_j[\nu_{jk}(V_{\varphi j}-V_{\varphi k})+\nu_{dj}V_{\varphi j}] &= n_j e_j E_\varphi^A + e_j B_\theta \hat{\Gamma}_{rj} + M_{\varphi j},\\
n_k m_k[\nu_{kj}(V_{\varphi k}-V_{\varphi j})+\nu_{dk}V_{\varphi k}] &= n_k e_k E_\varphi^A + e_k B_\theta \hat{\Gamma}_{rk} + M_{\varphi k},
\end{aligned}
\tag{7}
$$

where the first term on the left represents the interspecies friction force and the second term represents the viscous plus inertial plus charge-exchange forces, and the terms on the right represent the electric field, VXB, and external momentum input forces, respectively.

It would be natural to represent the IOL momentum input force for a two ion-species plasma by writing the toroidal momentum source as $M_{\varphi j} = M_{\varphi j}^{nbi} + M_{\varphi j}^{iol}$, and similarly for $M_{\varphi k}$, where the first term is the toroidal component of the neutral beam momentum input and the second term is an effective toroidal momentum input that would produce a difference in the solutions of Eqs. (7) with and without the $M_{\varphi j,k}^{iol}$ term equal to $\Delta V_{\varphi j}^{iol} = \Delta V_{\| j}(\mathbf{n}_\| \cdot \mathbf{n}_\varphi) = (2/\sqrt{\pi})M_{orbj} V_{thj}(\mathbf{n}_\| \cdot \mathbf{n}_\varphi)$ and similarly for $\Delta V_{\varphi k}^{iol}$.

Solving Eqs. (7) analytically, with and without the $M_{\varphi j,k}^{iol}$, for the difference in velocity, $\Delta V_{\varphi j,k}^{iol} = (2/\sqrt{\pi})M_{orbj} V_{thj}(\mathbf{n}_\| \cdot \mathbf{n}_\varphi)$, leads to expressions for the effective fluid toroidal momentum inputs that would produce this intrinsic toroidal rotation in the fluid solutions

$$
\begin{aligned}
M_{\varphi j}^{iol} &= n_j m_j[(\nu_{jk}+\nu_{dj})\Delta V_{\varphi j}^{iol} - \nu_{jk}\Delta V_{\varphi k}^{iol}],\\
M_{\varphi k}^{iol} &= n_k m_k[(\nu_{kj}+\nu_{dk})\Delta V_{\varphi k}^{iol} - \nu_{kj}\Delta V_{\varphi j}^{iol}].
\end{aligned}
\tag{8}
$$

We note that the form of Eq. (8) required in order for the effective fluid toroidal momentum inputs to represent IOL in fluid equations depends upon the details of the fluid equations and would be somewhat different for representations that differed from Eqs. (7).

The solutions to the fluid Eqs. (7) with the IOL momentum sources of Eqs. (8) yield the total toroidal velocities $\hat{V}_{\varphi j,k}$, i.e., the quantities that would be measured experimentally. Again, the carat indicates that the effects of ion orbit loss have been taken into account.

Deuterium (j) rotation velocity is not usually measured in tokamak experiments and a perturbation methodology[15] must be used to estimate it from the more commonly measured carbon (k) velocity. Using the above effective IOL momentum source to incorporate IOL effects directly in the fluid formalism enables this perturbation methodology to be applied directly, whereas treating the IOL effects in terms of the intrinsic rotation, as discussed in Sec. V, considerably complicates the perturbation formalism.

V. ADDITION OF INTRINSIC TOROIDAL ROTATION TO CALCULATED FLUID VELOCITY

Alternatively, Eqs. (6) can be solved for $M_{\varphi j,k} = M_{\varphi j,k}^{nbi}$ to obtain $V_{\varphi j,k}^{fluid}$ which does not take into account the presence of intrinsic rotation (but does take into account ion orbit particle loss effects on $\hat{\Gamma}_{rj,k}$). In this case, the total toroidal rotation that would be measured is understood to be

$$
\begin{aligned}
\hat{V}_{\varphi j,k}^{tot} &= V_{\varphi j,k}^{fluid} + V_{\varphi j,k}^{intrin} = V_{\varphi j,k}^{fluid} + \Delta V_{\varphi j,k}^{iol}\\
&= V_{\varphi j,k}^{fluid} + \Delta V_{\| j,k}(\mathbf{n}_\| \cdot \mathbf{n}_\varphi),\\
&= V_{\varphi j,k}^{fluid} + (2/\sqrt{\pi})M_{orbj}V_{thj,k}(\mathbf{n}_\| \cdot \mathbf{n}_\varphi).
\end{aligned}
\tag{9}
$$

Treating the experimental velocity as the superposition of the fluid velocity and the IOL kinetic velocity, as shown in Eq. (9), is equivalent to the effective IOL momentum formalism of Eq. (8). To verify this, we examine Eqs. (7) evaluated with total (experimental) velocities and insert the IOL external momentum source from Eqs. (8) and then collect terms to obtain

$$
\begin{aligned}
&n_j m_j\{\nu_{jk}[(\hat{V}_{\varphi j}^{tot} - \Delta V_{\varphi j}^{iol}) - (\hat{V}_{\varphi k}^{tot} - \Delta V_{\varphi k}^{iol})]\\
&\quad + \nu_{dj}(\hat{V}_{\varphi j}^{tot} - \Delta V_{\varphi j}^{iol})\} = B_\theta e_j \hat{\Gamma}_{rj} + n_j e_j E_\varphi^A + M_{\varphi j}^{nbi},\\
&n_k mk\{\nu_{kj}[(\hat{V}_{\varphi k}^{tot} - \Delta V_{\varphi k}^{iol}) - (\hat{V}_{\varphi j}^{tot} - \Delta V_{\varphi j}^{iol})]\\
&\quad + \nu_{dk}(\hat{V}_{\varphi k}^{tot} - \Delta V_{\varphi k}^{iol})\} = B_\theta e_k \hat{\Gamma}_{rj} + n_j e_j E_\varphi^A + M_{\varphi k}^{nbi}.
\end{aligned}
\tag{10}
$$

Equations (10) are identical to Eqs. (7) when Eqs. (8) are substituted into Eqs. (7). When intrinsic rotation is subtracted from the total rotation velocity, Eqs. (9) are shown to actually be evaluated with the fluid velocities obtained by solving Eqs. (7) with $M_{\varphi j,k} = M_{\varphi j,k}^{nbi}$ to obtain $V_{\varphi j,k}^{fluid}$. Therefore, it follows that correct inclusion of the IOL effects into the momentum balance equations can be accomplished either (1) by using the intrinsic rotation velocities to modify the fluid velocities or (2) by using an effective IOL momentum source to calculate modified fluid velocities.

VI. EFFECTIVE POLOIDAL MOMENTUM OF INTRINSIC ROTATION

The poloidal momentum balance equations (in the Kim-Diamond-Groebner[16] formulation of the extended Hirshman-Sigmar[17] model extended[18] to remove the trace impurity assumption of Ref. 16 and to include a poloidal momentum input, i.e., Eq. (9) of Ref. 17 with a momentum input) are

$$
[\nu_{viscj} + \nu_{jk}]\hat{V}_{\theta j} - [\nu_{jk}]\hat{V}_{\theta k} = \nu_{viscj}\frac{B_\varphi}{B^2}\frac{K^j}{e_j}L_{Tj}^{-1} + \frac{M_{\theta j}^{iol}}{n_j m_j} - \frac{e_j \hat{\Gamma}_{rj} B_\varphi}{n_j m_j}
\tag{11}
$$

and a similar equation with "j" and "k" interchanged. Here $\nu_{viscj} = \nu_{jj} B\mu_{00j}/B_\theta$ and the K^j and μ_{00} are Hirshman-Sigmar[17] coefficients defined explicitly in Ref. 18.

Solving Eqs. (11), with and without the $M^{iol}_{\theta j,k}$, the value of poloidal momentum input required to produce the difference in poloidal velocities, $\Delta\hat{V}^{iol}_{\theta j,k} = (2/\sqrt{\pi})M_{orbj,k} V_{thj,k}(\mathbf{n}_\parallel \cdot \mathbf{n}_\theta)$, leads to expressions for the effective fluid poloidal momentum inputs that would produce this intrinsic poloidal rotation in the fluid solution

$$M^{iol}_{\theta j} = n_j m_j[(\nu_{jk} + \nu_{viscj})\Delta\hat{V}^{iol}_{\theta j} - \nu_{jk}\Delta\hat{V}^{iol}_{\theta k}],$$
$$M^{iol}_{\theta k} = n_k m_k[(\nu_{kj} + \nu_{visck})\Delta\hat{V}^{iol}_{\theta k} - \nu_{kj}\Delta\hat{V}^{iol}_{\theta j}]. \quad (12)$$

The solutions to the fluid Eqs. (10) with the IOL momentum sources of Eqs. (12) yield the total poloidal momentum velocities, i.e., the quantities that would be measured experimentally.

VII. ADDITION OF INTRINSIC POLOIDAL ROTATION TO CALCULATED FLUID VELOCITY

Alternatively, Eqs. (11) can be solved with $M_{\theta j,k} = M^{nbi}_{\theta j,k} = 0$ to obtain $V^{fluid}_{\theta j,k}$ which does not take into account the presence of intrinsic rotation (but does take into account ion orbit particle loss effects on $\hat{\Gamma}_{rj,k}$). In this case, the total poloidal rotation that would be measured is understood to be

$$\hat{V}^{tot}_{\theta j,k} = V^{fluid}_{\theta j,k} + V^{intrin}_{\theta j,k} = V^{fluid}_{\theta j,k} + \Delta V_{\parallel j,k}(\mathbf{n}_\parallel \cdot \mathbf{n}_\theta)$$
$$= V^{fluid}_{\theta j,k} + (2/\sqrt{\pi})M_{orbj,k}V_{thj,k}(\mathbf{n}_\parallel \cdot \mathbf{n}_\theta) \quad (13)$$

VIII. CYLINDRICAL MODEL

A slab model has been used above in order to simplify the formalism somewhat to show more clearly how kinetic IOL effects can be included in fluid transport theory. While the slab model is approximately valid in the plasma edge where IOL is most important, it is straightforward to extend Eqs. (3) to cylindrical geometry by replacing the divergence terms on the left with $((1/r)\partial(r\hat{\Gamma}_{ri})/\partial r)$ and $((1/r)\partial(r\hat{Q}_{ri})/\partial r)$, which changes the solutions given by Eqs. (4) to

$$r\hat{\Gamma}_{ri}(r) = r_0\hat{\Gamma}_{ri}(r_0) + \int_{r_0}^{r} r' S_{ni}(r') e^{-2[F_{orbi}(r) - F_{orbi}(r')]} dr',$$
$$r\hat{Q}_{ri}(r) = r_0\hat{Q}_{ri}(r_0) + \int_{r_0}^{r} r' S_{Ei}(r') e^{-[E_{orbi}(r) - E_{orbi}(r')]} dr'. \quad (14)$$

IX. SUMMARY

We have defined how the kinetic ion orbit loss of particles, momentum, and energy from the outflowing distribution of thermalized ions in the edge of tokamak plasmas can be represented in a fluid transport theory, thus extending previous results[10] for inclusion of ion orbit loss particle and energy losses in fluid theory to also include the loss of toroidal and poloidal momentum associated with the intrinsic rotation velocities produced by ion orbit loss.

Another new result of this paper is the identification of poloidal intrinsic rotation and an effective poloidal momentum source due to IOL. Although it follows naturally from the above development, this potentially important new phenomenon does not seem to have been previously observed or predicted, as far as we are aware.

[1] W. M. Stacey, Phys. Plasmas **18**, 102504 (2011).
[2] W. M. Stacey and M. T. Schumann, Phys. Plasmas **22**, 042504 (2015).
[3] J. S. deGrassie, R. J. Groebner, K. H. Burrell, and W. M. Solomon, Nucl. Fusion **49**, 085020 (2009).
[4] J. S. deGrassie, J. E. Rice, K. H. Burrell, R. J. Groebner, and W. M. Solomon, Phys. Plasmas **14**, 056115 (2006).
[5] W. M. Stacey, J. A. Boedo, T. E. Evans, B. A. Grierson, and R. J. Groebner, Phys. Plasmas **19**, 112503 (2012).
[6] W. M. Stacey, M.-H. Sayer, J.-P. Floyd, and R. J. Groebner, Phys. Plasmas **20**, 012509 (2013).
[7] W. M. Stacey and B. A. Grierson, Nucl. Fusion **54**, 073021 (2014).
[8] C. Pan, S. Wang, and J. Ou, Nucl. Fusion **54**, 103003 (2014).
[9] W. M. Stacey, Nucl. Fusion **55**, 058001 (2015).
[10] W. M. Stacey, Phys. Plasmas **21**, 014502 (2014).
[11] A. V. Chankin and G. M. McCracken, Nucl. Fusion **33**, 1459 (1993).
[12] P. Helander and R. J. Akers, Phys. Plasmas **12**, 112503 (2005).
[13] W. M. Stacey, Nucl. Fusion **53**, 063011 (2013).
[14] W. M. Stacey, Contrib. Plasma Phys. **48**, 94 (2008).
[15] W. M. Stacey and R. J. Groebner, Phys. Plasmas **15**, 012503 (2008).
[16] B. Kim, P. H. Diamond, and R. J. Groebner, Phys. Fluids B **3**, 2050 (1991).
[17] S. P. Hirshman and D. J. Sigmar, Nucl. Fusion **21**, 1079 (1981).
[18] W. M. Stacey, Phys. Plasmas **15**, 012501 (2008).