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1985 Nucl. Fusion 25 463

(http://iopscience.iop.org/0029-5515/25/4/007)

ROTATION AND IMPURITY TRANSPORT IN A TOKAMAK PLASMA WITH DIRECTED NEUTRAL-BEAM INJECTION

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ABSTRACT. The authors have extended their previous collisional-regime theory for rotation and impurity transport in a tokamak plasma with strong, directed NBI and strong rotation ($v_\phi \approx v_{th}$) to the plateau regime. The paper gives a summary of a kinetic theory derivation of the parallel viscous force in the strong rotation ordering and a self-consistent formalism for calculating ion and impurity rotation velocities and radial transport fluxes, as well as the radial electric field and the poloidal variation of the impurity density upon which the former strongly depend. Calculations for model problems representative of ISX-B and PLT are presented. The predicted impurity transport exhibits features that are in agreement with experimental observations.

1. INTRODUCTION

There is a long-standing interest in the possibility of using directed neutral-beam injection (NBI) to reverse the normal inward diffusion of impurities in tokamak plasmas. Ohkawa [1] noted that the direct momentum exchange of injected beam particles and impurities via collisions would produce a radial impurity transport flux and predicted that counter-injection would produce an outward impurity transport flux.

Stacey and Sigmar [2] noted that the injected beam momentum must be balanced by a radial transfer of momentum, or drag, and that this allowed a unique determination of the radial electric field. They predicted that when the effect of the momentum input and drag on the particle flows was taken into account and when the effect of the radial electric field on transport was treated self-consistently, then co-injection would produce an outward impurity flux.

Burrell et al. [3] pointed out that the large toroidal rotation velocities associated with directed NBI could produce poloidal non-uniformity in the impurity density over the flux surface, which in turn could produce a radial impurity transport flux. This effect becomes significant when the impurity rotation velocity becomes comparable with its thermal velocity. They predicted that co-injection would produce an outward impurity flux due to this rotation, or inertial, effect. However, this theory was not self-consistent with respect to the ambipolar electric field.

Recently, Stacey and Sigmar [4] extended their previous formalism [2] to include this rotation effect in a self-consistent theory for particle flows in the flux surface, the radial electric field and radial particle transport in a tokamak plasma in the collisional regime with directed NBI. A more extensive discussion of relevant previous work is also given in Ref. [4].

Experimentally, it has been observed in PLT [5, 6] and ISX-B [7, 8] that the central accumulation of edge-introduced impurities is much greater with counter-injection than with co-injection, in qualitative agreement with the more recent theory. Attempts [8, 9] to quantitatively interpret some of the experimental results with earlier, incomplete versions [2, 3] of the theory have been encouraging, although they have been unable to explain all features of both the co-injected and counter-injected results.

The purpose of this paper is to extend our previous theory [4] for the collisional regime to the plateau regime. This extension accommodates the important case of the main ion species being nearly collisionless and the impurity species being collisional, which we refer to as the mixed regime. This extension requires a kinetic theory solution for the parallel viscous force, $B \cdot \nabla \cdot \overleftrightarrow{\pi}$, in the presence of large rotation velocities and an incorporation of this viscous force into the fluid theory. We find that in the mixed collisionality regime the mechanisms dominating the transport processes are quite different from those in the collisional regime.

The theory described in this paper provides a self-consistent (non-linear) model, based upon particle and momentum conservation and charge neutrality, for calculating toroidal and poloidal rotation velocities, the radial electric field and radial particle fluxes in a two-species (ion-impurity) tokamak plasma with strong, directed neutral-beam injection, and the resulting large rotation velocities. The theory relies upon neoclassical theory for specification of the parallel viscous force (including the plateau resonance) but allows for an anomalous viscous radial transfer of toroidal momentum as indicated by experimental data [7, 10]. Prescriptions are given for determining the anomalous radial momentum transfer rates from measured rotation velocities for the ions and impurities.

The paper is organized as follows. In Section 2, the derivation of the parallel viscous force for plateau regime ions from kinetic theory is briefly outlined, for a strongly rotating ($v_\phi \approx v_{th}$) plasma, including the density variation over the flux surface. With this constitutive relation for the viscous force in hand, a fluid formalism is developed in Section 3 for the rotation velocities and particle transport in a two-species (ion-impurity) plasma in the strong rotation ordering. In Section 4, the formalism is applied to plasmas with the gross features of ISX-B and PLT in order to predict certain features that are observed in the experiments.

2. PARALLEL VISCOUS FORCE IN THE PLATEAU REGIME

An essential step in extending our previous collisional regime theory to the mixed regime is the calculation of the parallel viscous force in the presence of large toroidal rotation velocities, which derivation we briefly outline. Following Shaing and Callen [11] we write

$$\vec{B}\cdot\nabla\cdot\overleftrightarrow{\pi} \equiv \int d^3v\ \tfrac{1}{2}\, mv^2\ (\hat{n}\cdot\nabla B) f_1 \qquad (1)$$

where $\hat{n} = \vec{B}/B$, and the perturbed distribution function f_1 is the solution of

$$D(f_1) - C(f_1) = v'_\parallel \frac{\partial}{\partial\theta}\left(\frac{Iv'_\parallel}{\Omega}\right)\Bigg|_{E,\mu}\left(\frac{1}{JB}\frac{\partial f_0}{\partial\psi}\right) \qquad (2)$$

where D is the streaming operator, C is the collision operator, J is the Jacobian, $I = RB_\phi$, Ω is the gyroradius, and f_0 is the Maxwellian distribution function which is shifted owing to rotation. Shaing and Sigmar [12] solved Eq.(2) and showed that in the strong rotation case

$$f_0 = n\left(\frac{m}{2\pi T}\right)^{3/2} \exp\left[-\frac{m}{2T}\left(v'^2 + v'^2_\parallel\right) - \frac{e\tilde{\Phi}}{T} + \frac{m}{2T}\tilde{u}^2\right] \qquad (3)$$

Here, n and T are the density and temperature components which are uniform over the flux surface, $v'_\parallel = v_\parallel - u$, Φ is the electrostatic potential,

$$u \equiv -\left(\frac{I}{B}\right)\frac{\partial}{\partial\psi}\Phi(\psi,\theta) \qquad (4)$$

is the parallel flow due to the radial electric field, and the poloidally varying components are

$$\tilde{\Phi} \equiv \Phi - \langle\Phi\rangle \qquad \tilde{u}^2 = u^2 - \langle u^2\rangle \qquad (5)$$

where $\langle\,\rangle$ denotes a flux surface average. Note that this expression for f_0 contains the term $(m/2T)\tilde{u}^2$, which corresponds to the inertial term of the fluid theory (presented in Section 3).

To solve Eq.(2), let

$$f_1 = -\frac{Iv'_\parallel}{\Omega}\frac{\partial f_0}{\partial\psi} + \frac{v'_\parallel}{v} S(\psi, v') \frac{B}{\sqrt{\langle B^2\rangle}} f_0 + h_1 \qquad (6)$$

where S is a smooth function of energy, and h_1 is a localized function of pitch angle, which is due to the plateau resonance D−C = 0. For a given S, the equation for h_1 is

$$(D-C)h_1 = \tfrac{1}{2} v' \frac{(\hat{n}\cdot\nabla B)}{\sqrt{\langle B^2\rangle}} Sf_0 \qquad (7)$$

For circular flux surfaces, with minor radius r and major radius R, where $B_p = B_p^0(r)/(1+\epsilon\cos\theta)$, and $\hat{n}\cdot\nabla B = (B_p^0/r)\epsilon\sin\theta$ ($\epsilon \equiv r/R$), Eq.(7) has the solution

$$h_1 = \tfrac{1}{2}\epsilon S\left(\frac{\nu_s}{\omega_t}\right)^{-1/3} \frac{B}{\sqrt{\langle B^2\rangle}} f_0 \int_0^\infty \sin(\theta - p\tau)\exp(-\tau^3/6)d\tau \qquad (8)$$

where the resonance integral has the property

$$\int_{-1}^{1} d\lambda' \int_{0}^{\infty} \cos(p\tau)\exp(-\tau^3/6)d\tau = \pi(\nu_s/\omega_t)^{1/3} \qquad (9)$$

Here, λ is the pitch-angle variable, $d^3v = 2\pi v^2 dv d\lambda$; ν_s is the collisional pitch-angle scattering frequency; $\omega_t = v_{th}/Rq$ is the transit frequency; q is the safety factor; and $p \equiv (\nu_s/\omega_t)^{-1/3}(v'_\parallel/v')$ is normalized such that p = 1 for resonant particles.

Finally, the driving term $S(\psi, v')$ can be determined self-consistently from the $v_\parallel L_0$ and $v_\parallel L_1$ moments of Eq.(6), using the expansion

$$S(\psi, v') = \frac{2v'}{v_{th}}\ (A_0L_0 + A_1L_1 + \ldots) \qquad (10)$$

where the Sonine polynomials $L_0 = 1$, $L_1 = \frac{5}{2} - (v')^2/v_{th}^2$, etc. In this case, the term h_1, which is localized in pitch angle, can be neglected. We thus obtain expressions for A_0 and A_2 in terms of $v_\parallel$ and $q_\parallel$, the parallel particle and heat flows. These flows are related through the fluid equations to the radial pressure gradient [11].

Using Eqs (3), (6), (8) and (10) in Eq.(1) yields the final result for the parallel viscous force

$$\vec{B}\cdot\nabla\cdot\overleftrightarrow{\pi} = \langle\vec{B}\cdot\nabla\cdot\overleftrightarrow{\pi}\rangle\, 2\sin^2\theta \qquad (11)$$

The flux surface averaged viscous force is

$$\langle\vec{B}\cdot\nabla\cdot\overleftrightarrow{\pi}\rangle = 3\langle(\hat{n}\cdot\nabla B)^2\rangle\, \mu'_1\, \vec{v}\cdot\vec{B}_p/B_p^2 \qquad (12)$$

where, for species j,

$$\mu'_{1j} = \mu_{1j}g_j \equiv \left[\frac{Rqn_jm_jv_{thj}\nu_{*j}}{(1+\nu_{*j})(1+\epsilon^{3/2}\nu_{*j})}\right] g_j \qquad (13)$$

and the strong rotation correction factor is

$$g_j = \frac{2}{3}\sqrt{\pi}\left[1 + \left(\frac{v_{\phi j}}{v_{thj}}\right)^2\left(1 - \frac{e_j}{m_j}\Psi\right)\right] \qquad (14)$$

with the pressure $p_j = n_jT_j = \frac{1}{2}n_jm_jv_{th}^2$, e_j being the charge, m_j being the mass, $\nu_{*j} = \nu_{jj}qR/\epsilon^{3/2}v_{thj}$, and

$$\Psi \equiv \left[\sum_j (n_je_jm_j)/T_j\right]\left[\sum_j (n_je_j^2)/T_j\right]^{-1} \qquad (15)$$

For circular flux surfaces

$$\langle(\hat{n}\cdot\nabla B)^2\rangle = \frac{1}{2}(\epsilon/Rq)^2(B^0)^2 \qquad (16)$$

We will see in the next section that the parallel viscous force of Eq.(11) enters into the determination of the poloidal variation of the densities over the flux surface, and the flux surface averaged parallel viscous force of Eq.(12) enters into the determination of the rotation velocities. We now examine the conditions under which the unaveraged force of Eq.(11) must be retained in the calculation, in the plateau regime, i.e. when it is significant relative to the parallel pressure gradient.

$$0 \gtrsim -T_j\vec{B}\cdot\nabla n_j - \vec{B}\cdot\nabla\cdot\overleftrightarrow{\pi}_j \qquad (17)$$

or, with $\vec{B}\cdot\nabla = \frac{B_p}{r}\frac{\partial}{\partial\theta}$, and the ordering

$$\frac{1}{n_j}\frac{\partial n_j}{\partial\theta} \approx \left(\frac{v_{\phi j}}{v_{thj}}\right)^2 \epsilon$$

$$\frac{r}{B_p}\ \vec{B}\cdot\nabla\cdot\overleftrightarrow{\pi}_j/n_jT_j \gtrsim \epsilon\left(\frac{v_{\phi j}}{v_{thj}}\right)^2$$

Making use of Eq.(11), this becomes

$$\left(\frac{B_\phi}{B_p}\right)\left(\frac{v_{pj}}{v_{thj}}\right) \gtrsim \left(\frac{v_{\phi j}}{v_{thj}}\right)^2 \Big/ \epsilon\left[1 + \left(\frac{v_{\phi j}}{v_{thj}}\right)^2\left(1 - \frac{e_j}{m_j}\Psi\right)\right] \qquad (18)$$

where the factor in square brackets is O(2) in the strong rotation ordering.

We note that in a strongly beam-driven plasma, Eq.(18) is essentially a condition on the beam momentum input (see Eqs (26) and (28)).

We find that the condition (18) is satisfied only at the largest magnitudes of rotation ($v_\phi^2 \gtrsim v_{th}^2$) (as can be seen from Fig.1, noting that for ISX-B, $v_{thi} \gtrsim 1.5 \times 10^5\ m\cdot s^{-1}$). When this condition is satisfied, the $\cos 2\theta$ viscosity-driven variation of Eq.(11) is important in driving the poloidal variation in the main ion density and the electrostatic potential.

In contrast, the centrifugal-driven variation is proportional to (see Eq.(3))

$$\exp\left\{\frac{m}{2T}\ \tilde{u}^2\right\} = \exp\left\{\left(\frac{v_\phi}{v_{th}}\right)^2 (B^0)^2\left(\frac{1}{B^2} - \left\langle\frac{1}{B^2}\right\rangle\right)\right\}$$

where the geometrical dependence is $\epsilon\cos\theta$. Note that even for $(v_\phi/v_{th})^2 \gtrsim 1$ and $\epsilon < 1$, this expansion has rapidly decaying higher harmonics.

3. FLUID THEORY

3.1. Outline of the derivation

The construction of the expressions for the radial electric field, the particle flows within the flux surface, the poloidal variation of the particle densities over the flux surface and the transport fluxes across the flux surfaces closely parallels that of Ref. [4] (treating the collisional regime) to which we refer the reader interested in details. In this section, we outline the derivation, and in subsequent sections we present and discuss the new results for the mixed regime.

The basic equations are the particle continuity equation for species j

$$\nabla \cdot n_j \vec{v}_j = 0 \tag{19}$$

the momentum balance equation for species j

$$n_j m_j (\vec{v}_j \cdot \nabla)\vec{v}_j + \nabla p_j + \nabla \cdot \overleftrightarrow{\pi}_j = -n_j e_j \nabla\Phi + n_j e_j (\vec{v}_j \times \vec{B}) + \vec{R}_j + \vec{N}_j \tag{20}$$

and charge neutrality

$$\sum_{j=1}^{J} n_j e_j = 0 \tag{21}$$

In Eq.(20), R_j is the interspecies friction, which is represented by

$$\vec{R}_j = -n_j m_j \sum_{k \neq j} \nu_{jk} (\vec{v}_j - \vec{v}_k) \tag{22}$$

and

$$\vec{N}_j = \vec{M}_j - n_j m_j \nu_{dj} \vec{v}_j \tag{23}$$

represents 'external' momentum exchange of particles of species j, which is due to momentum input from collisions with fast ions from neutral-beam injection $\vec{M}_j$ and to the 'radial' transfer of momentum across the flux surface (due to anomalous viscous effects, as suggested by experiment [7, 10]), represented by a drag frequency, ν_{dj}. Stacey and Sigmar [13] have recently shown that the toroidal neoclassical viscous force on each species can be represented in this form in terms of a drag frequency. The parallel neoclassical viscous forces are contained in the term $\nabla \cdot \overleftrightarrow{\pi}_j$.

In these equations, n, m, e and $\vec{v}$ refer to the particle density, mass, charge and flow velocity of particle species j; p is the pressure and π is the anisotropic stress tensor discussed in Section 2; Φ is the electrostatic potential; $\vec{B}$ is the magnetic field; ν_{jk} is the collision frequency between particle species j and k.

Equation (19) and the perpendicular (in the flux surface) component of Eq.(20) for each species can be solved, to within a constant of integration (which is proportional to the poloidal flow velocity), for the lowest-order (in the gyroradius) particle flows, which lie in the flux surface. The equation obtained by summing over the species in the flux surface averaged toroidal component of Eq.(20) can then be solved for the 'radial' electric field. The constants of integration mentioned previously can be found by solving simultaneously the flux surface averaged parallel components of Eq.(20) for all species. The 'poloidal' variation of the particle densities and electrostatic potential can then be obtained by solving the parallel (to B) components of Eq.(20), subject to the constraint of Eq.(21). Finally, the transport flux of particles across the flux surface can be obtained by combining the parallel and normal (to the flux surface) components of Eq.(20).

At this point, we specialize our results to a plasma constituted of a main ion species (i), an impurity species (I) and electrons (e), in which ion/impurity collisions are dominant over ion/electron or impurity/electron collisions in determining the transport of ions and impurities ($\alpha \equiv n_I z^2/n_i \gg \sqrt{m_e/m_i}$). Actually, $\alpha \gg \sqrt{m_e/m_i}$ is only a limit on the regime of validity for the theory for the main ion species. The theory is valid for the impurity species down to trace impurity concentrations ($\alpha \to 0$). For the plasma equilibrium, we make the large-aspect-ratio, circular-ψ, low-beta approximation. We use the subscripts ϕ and p to refer to toroidal and poloidal components, and we use the subscript 0 to denote the component that is uniform over the flux surface. A number of parameters which arise in the derivation are now defined:

$$P_j' \equiv \frac{1}{n_j^0 e_j B_p^0} \frac{\partial p_j^0}{\partial r}$$

$$\nu_{j*} \equiv \frac{\nu_{jj}\, q R_0}{\epsilon^{3/2} v_{thj}}$$

$$v_{thj} \equiv \frac{T}{m_j} \qquad j = i, I$$

The normalized drag frequencies are

$$\beta_i \equiv \frac{\nu_{di}}{\nu_{iI}} \qquad \beta_I \equiv \frac{\nu_{dI}}{\nu_{Ii}}$$

(where $n_i m_i \nu_{iI} = n_I m_I \nu_{Ii}$ from momentum conservation).

The normalized viscosities are

$$\hat{\mu}_j \equiv \frac{\frac{3}{2}\sqrt{\epsilon}\, f_j h_j}{(1+\nu_{*j})(1+\epsilon^{3/2}\nu_{*j})} \qquad j = i, I$$

where

$$f_i \equiv \sqrt{2}/\alpha$$

$$f_I \equiv \alpha\sqrt{\frac{m_I}{2m_i}}$$

$$\ell \equiv \frac{m_I}{z m_i}$$

$$\alpha \equiv \frac{n_I^0 z^2}{n_i^0}$$

$$h_i \equiv 1 + \frac{\alpha(1-\ell)}{1+\alpha}\left(\frac{v_{\phi i}}{v_{thi}}\right)^2$$

$$h_I \equiv 1 + \frac{(\ell-1)}{\ell(1+\alpha)}\left(\frac{v_{\phi I}}{v_{thI}}\right)^2$$

$$\xi_j \equiv \hat{\mu}_j + \beta_j$$

$$M_{xj} \equiv \hat{n}_x \cdot \int d^3v\, \vec{v} m_j C_{bj} \qquad j = i, I \tag{24}$$

In the above definitions, R_0 is the major radius, $\epsilon = r/R_0$ is the inverse aspect ratio, T is the temperature, q is the safety factor, ν_{jj} is the self-collision frequency, C_{bj} is the Fokker-Planck collision operator, and $\hat{n}_x$ is the unit vector in the x-direction (e.g. parallel, toroidal). Throughout the paper, the 'hat' notation will signify either a unit vector or the normalization

$$\hat{A}_i \equiv \frac{A_i}{n_j^0 m_j \nu_{iI}^0}$$

$$\hat{A}_I \equiv \frac{A_I}{n_I^0 m_I \nu_{Ii}^0}$$

3.2. Comparison with collisional regime formalism

The formalism presented in this section differs in several significant ways from the analogous formalism that was previously given [4] for the case where the bulk ions and impurities are in the collisional regime. An explicit expression for the radial electric field (Eq.(25)) is obtained when the main ion viscous force term is present, whereas a coupled set of non-linear equations, with multiple roots, must be solved to obtain the radial electric field and the poloidal density variation in the collisional regime (Eqs (23)–(26) and (29)–(31) of Ref. [4]). This result obtains because the viscous force is of lower order in ϵ than the inertial force. The presence of the parallel viscous force modifies the expressions for the toroidal rotation velocities and alters fundamentally the relationship between the impurity and main ion poloidal rotation velocities (compare Eq.(29) with Eqs (37) and (38) of Ref. [4]). The presence of the parallel viscous force leads to a new component of the radial transport flux (labelled NC in Eq.(39)) and modifies significantly the other components of the transport flux (there are now terms $\sim\mu$ from the viscous force term, whereas in Ref. [4] all terms are $\sim\epsilon^2$).

3.3. Radial electric field

The component of the radial electric field (E_r) which is constant over the flux surface is obtained self-consistently from the flux surface averaged toroidal momentum balance equations summed over species.

$$\begin{aligned}\frac{E_r^0}{B_p^0} = {} & [\{\hat{\mu}_i + \hat{\mu}_I(1+\xi_i)\}\hat{M}_{\phi I}^0 + \{\hat{\mu}_I + \hat{\mu}_i(1+\xi_I)\} \\ & \times M_{\phi i}^0 + \{\beta_i + \beta_I(1+\xi_i)\}\hat{\mu}_I P_I' \\ & + \{\beta_I + \beta_i(1+\xi_I)\}\hat{\mu}_i P_i'][\hat{\mu}_i\{\beta_I + \beta_i(1+\xi_I)\} \\ & + \hat{\mu}_I\{\beta_i + \beta_I(1+\xi_i)\}]^{-1}\end{aligned} \tag{25}$$

This result is the same as the one obtained [2] in the weak rotation ($v_\phi \ll v_{th}$) ordering.

Examination of this expression reveals several points. The radial electric field scales as $E_r \approx \hat{M}/\beta \approx M/\nu_d$, the ratio of the NBI momentum input to the radial momentum transport or 'drag' frequency, and is relatively insensitive to the parallel neoclassical viscosity coefficients $\hat{\mu}_j$. Neutral-beam co-injection ($M > 0$) contributes a positive component to the radial electric field ($E_r > 0$), and neutral-beam counter-injection contributes a negative component. The normal negative main ion pressure gradient ($P_i' < 0$) produces a negative contribution ($\Delta E_r < 0$) to the radial electric field, and similarly for the impurity pressure gradient, although the latter is usually unimportant because $|P_I'| \cong 1/z\,|P_i'| \ll |P_i'|$.

3.4. Rotation velocities

Equation (19) and the perpendicular and flux surface averaged parallel components of Eq.(20) can be used to determine the toroidal and poloidal rotation velocities.

The toroidal component of the impurity rotation velocity is

$$v_{I\phi}^0 = \frac{[\hat{M}_{\|i}^0 + (1+\xi_i)\hat{M}_{\|I}^0] - [\hat{\mu}_I(1+\xi_i)P_I' + \hat{\mu}_i P_i'] + [\hat{\mu}_I(1+\xi_i) + \hat{\mu}_i]\left(\frac{E_r^0}{B_p^0}\right)}{\xi_i(1+\xi_I) + \xi_I} \tag{26}$$

This equation provides a prescription for determining the radial electric field, E_r, from the measured impurity rotation velocity, measured pressure gradients and calculated NBI momentum inputs. Equation (25) can be used to eliminate the radial electric field, leading to

$$v_{I\phi}^0 = \frac{[\{\hat{\mu}_I + \hat{\mu}_i(1+\hat{\mu}_I)\}\hat{M}_{\|i}^0 + \{\hat{\mu}_i(1+\beta_i) + \hat{\mu}_I(1+\xi_i)\}\hat{M}_{\|I}^0] + [\hat{\mu}_i\hat{\mu}_I\beta_i(P_i' - P_I')]}{\hat{\mu}_i\{\beta_I + \beta_i(1+\xi_I)\} + \hat{\mu}_I\{\beta_i + \beta_I(1+\xi_i)\}} \tag{26'}$$

The toroidal component of the main ion rotation velocity is obtained by exchanging the i and I subscripts.

Examination of these expressions reveals several interesting points. The magnitude of the toroidal rotation velocity scales as $v_\phi \approx \hat{M}/\beta \approx M/\nu_d$ and is relatively insensitive to the parallel neoclassical viscosity coefficients $\hat{\mu}$. The toroidal rotation is in the direction of the NBI momentum injection, except for the effect of the radial pressure gradients. The usual negative main ion pressure gradient ($P_i' < 0$) produces a negative contribution to the impurity toroidal rotation ($\Delta v_{I\phi} > 0$) and a positive contribution to the main ion toroidal rotation ($\Delta v_{i\phi} < 0$), which persist even in the absence of NBI. ($v_\phi > 0$ corresponds to rotation in the positive ϕ-direction in a ψ-p-ϕ co-ordinate system in which the toroidal magnetic field is in the positive ϕ-direction.)

In general, the main ions and impurities do not have a common toroidal rotation velocity, because of the different momentum input rates and radial momentum transfer rates. The difference in rotation velocities is given by

$$v_{i\phi}^0 - v_{I\phi}^0 = \frac{(\hat{\mu}_i + \hat{\mu}_I)(\beta_I\hat{M}_{\|i}^0 - \beta_i\hat{M}_{\|I}^0) - \hat{\mu}_i\hat{\mu}_I(\beta_i + \beta_I)(P_i' - P_I')}{\hat{\mu}_i\{\beta_I + \beta_i(1+\xi_I)\} + \hat{\mu}_I\{\beta_i + \beta_I(1+\xi_i)\}} \tag{27}$$

In general, both the main ions and impurities will rotate toroidally in the direction of the NBI momentum input, but the main ions will rotate faster because $\beta_I \cong \beta_i$ and $|M_i| > |M_I|$, as discussed later. The usual negative main ion pressure gradient ($P_i' < 0$) produces a positive contribution to $v_{i\phi} - v_{I\phi}$, which increases the difference in toroidal rotation for co-injection and decreases it for counter-injection.

Physically, the difference between ion and impurity rotation velocities may be understood as follows. Momentum is input to each species from the beam, which accelerates each species. Momentum is transferred between species (friction) and across flux surfaces (drag). Friction tends to reduce the velocity difference between species. However, if the drag force is larger on the impurities than on the main ions (see Section 3.5) then the main ions will accelerate to a larger velocity than the impurities.

The poloidal rotation velocity of the impurities is given, to leading order, by

$$v_{Ip}^0 = -\left(\frac{B_p^0}{B_\phi^0}\right)\frac{\hat{\mu}_i[(\beta_I\hat{M}_i^0 - \beta_i\hat{M}_I^0) - (\beta_i + \beta_I(1+\beta_i))(P_I' - P_i')]}{\hat{\mu}_i\{\beta_I + \beta_i(1+\xi_I)\} + \hat{\mu}_I\{(\beta_i + \beta_I(1+\xi_i)\}} \tag{28}$$

and the poloidal rotation velocity of the main ions is obtained by exchanging the i and I subscripts. It is interesting to note that

$$v_{ip}^0 = -\frac{\hat{\mu}_I}{\hat{\mu}_i} v_{Ip}^0 \tag{29}$$

which result can be obtained directly by summing the parallel components of Eq.(20) over species and using Eq.(12), i.e.

$$\sum_j \langle B \cdot \nabla \cdot \overleftrightarrow{\pi}_j \rangle = \mu_I v_{Ip}^0 + \mu_i v_{ip}^0 + O(\epsilon^2)$$

Examination of Eq.(28) reveals that usually the poloidal rotation of the main ions will be positive and the poloidal rotation of the impurities will be negative for co-injection, and conversely for counter-injection. (Here, 'positive' means the positive θ-direction in a right-hand r-θ-ϕ toroidal co-ordinate system in which

B is in the positive ϕ-direction.) The usual negative main ion pressure gradient ($P_i' < 0$) produces a negative contribution to the main ion poloidal rotation and a positive contribution to the impurity poloidal rotation, which persist in the absence of neutral-beam injection. This result of oppositely directed poloidal rotation velocities arises because viscous forces dominate the parallel momentum balance; this is quite different from the result [4] in the collisional regime, where the viscous forces are subdominant.

3.5. Determination of drag frequencies

Equation (26) and the corresponding equation for the main ions (which is obtained by exchanging i and I subscripts) provide a means for experimentally determining the anomalous viscous drag coefficients ν_{di} and ν_{dI} in terms of measured toroidal rotation velocities of the main ions and impurities. Equation (28) and the corresponding equation for the main ions could also be used to determine the drag coefficients in terms of measured poloidal rotation velocities for the two species

$$\beta_i \equiv \frac{\nu_{di}}{\nu_{iI}} = \left(\frac{M_{\phi i}^0}{n_i m_i \nu_{iI} v_{i\phi}^0}\right) + \left(1 - \frac{v_{I\phi}^0}{v_{i\phi}^0}\right) \tag{30}$$

$$\beta_I \equiv \frac{\nu_{dI}}{\nu_{Ii}} = \left(\frac{M_{\phi I}^0}{n_I m_I \nu_{Ii} v_{I\phi}^0}\right) + \left(\frac{v_{i\phi}^0}{v_{I\phi}^0} - 1\right) \tag{31}$$

Note that measurements of the toroidal and poloidal rotation velocities for one species do not serve to determine the drag coefficients for both species because the toroidal and poloidal rotation velocities are uniquely related by the radial momentum balance (i.e. the radial component of Eq.(20))

$$v_{Ip}^0 = \left(\frac{B_p^0}{B_\phi^0}\right)\left[v_{I\phi}^0 + P_I' - \frac{E_r^0}{B_p^0}\right] \tag{32}$$

and a similar equation for v_{ip}, with I replaced by i.

The definition of β leads to

$$\frac{\beta_I}{\beta_i} = \frac{\nu_{dI}}{\nu_{di}} \cdot \frac{\nu_{iI}^0}{\nu_{Ii}^0}$$

Although the process which produces the anomalous radial transfer of viscosity is not understood, it is plausible that the actual transfer of momentum involves collisions. This leads us to assume

$$\frac{\nu_{dI}}{\nu_{di}} = \frac{\nu_{II}^0 + \nu_{Ii}^0}{\nu_{ii}^0 + \nu_{iI}^0} = \alpha z^2 \sqrt{\frac{m_i}{m_I}} \left(\frac{1 + \sqrt{\frac{2}{\alpha}}\sqrt{\frac{m_I}{m_i}}}{1 + \alpha\sqrt{2}}\right) \frac{\ln \Lambda_I}{\ln \Lambda_i}$$

from which we obtain the relation

$$\frac{\beta_I}{\beta_i} = \alpha^2 \sqrt{\frac{m_I}{m_i}} \left(\frac{1 + \sqrt{2/\alpha}\sqrt{\frac{m_I}{m_i}}}{1 + \alpha\sqrt{2}}\right)\left(\frac{\ln \Lambda_I}{\ln \Lambda_i}\right) \tag{33}$$

where we have used $n_i m_i \nu_{iI} = n_I m_I \nu_{Ii}$ from momentum balance.

3.6. Density and potential variation over the flux surface

The parallel components of the momentum balance of Eq.(20) for the main ions, impurities and electrons and the charge neutrality condition of Eq.(21) can be solved for the poloidal variation of n_i, n_I, n_e and Φ over the flux surface. The solution is carried out by expanding each of these variables in the form

$$x(r,\theta) = x^0(r)\,(1 + x^c \cos\theta + x^s \sin\theta)$$

and then taking the $\sin\theta$ and $\cos\theta$ moments (over θ) of the above-mentioned equations to obtain algebraic equations for the unknowns, n_I^s, n_I^c, etc. Since the $\sin\theta$ and $\cos\theta$ moments of $\vec{B}\cdot\nabla\cdot\overleftrightarrow{\pi}$ vanish (see Eq.(11)), the resulting equations are exactly the same as those given previously [4] for the collisional case. We note that

$$\nu_{ik} = \nu_{ik}^0\left(\frac{n_k}{n_k^0}\right)$$

$$M_j = M_j^0\left(\frac{n_j}{n_j^0}\right)$$

In most plasmas of interest, $n_I z/n_i \equiv \alpha/z \ll 1$, in which case the $\sin\theta$ and $\cos\theta$ moment equations can be reduced and solved directly for n_I^c and n_I^s

$$\left(\frac{n_I^c}{\epsilon}\right) = \frac{\left(\frac{1}{qR_0\nu_{Ii}^0}\right)[((1+\frac{1}{2}\alpha)v_{thI}^2 - Y^2)(Y^2+\omega^2)] - [(1+\beta_I)Y(\hat{M}_\phi^0 + 2(P_I' - P_i' + \beta_I\omega))]}{[(1+\beta_I)Y]^2 + \left|\left(\frac{1}{qR_0\nu_{Ii}^0}\right)\left((1+\frac{1}{2}\alpha)v_{thI}^2 - Y^2\right)\right|^2} \tag{34}$$

$$\left(\frac{n_I^s}{\epsilon}\right) = \frac{\left(\frac{1}{qR_0\nu_{Ii}^0}\right)[(1+\beta_I)Y(Y^2+\omega^2) + ((1+\frac{1}{2}\alpha)v_{thI}^2 - Y^2)(\hat{M}_{\phi I}^0 + 2(P_I' - P_i' + \beta_I\omega))]}{[(1+\beta_I)Y]^2 + \left|\left(\frac{1}{qR_0\nu_{Ii}^0}\right)\left((1+\frac{1}{2}\alpha)v_{thI}^2 - Y^2\right)\right|^2} \tag{35}$$

where

$$Y \equiv \left(\frac{B_\phi^0}{B_p^0}\right) v_{Ip}^0 \left(\frac{\beta_I + \beta_i (1+\xi_I)}{\beta_I + \beta_i (1+\beta_I)}\right) \cong \left(\frac{B_\phi^0}{B_p^0}\right) v_{Ip}^0$$

$$\omega \equiv P_I' - \frac{E_r^0}{B_p^0} = \left(\frac{B_\phi^0}{B_p^0}\right) v_{Ip}^0 - v_{I\phi}^0 \qquad (36)$$

Equations (34) and (35) reveal several noteworthy points. The sin-component $n_I^s \approx Y \approx v_{Ip}^0$ in the absence of pressure gradients, in which case n_I^s would be negative for co-injection, positive for counter-injection and an antisymmetric function of beam power. The usual negative main ion pressure gradient ($P_i' < 0$) produces a positive contribution to n_I^s, which would increase the magnitude of the positive n_I^s for counter-injection and decrease the magnitude of the negative n_I^s for co-injection. In our co-ordinate system, θ is measured counter-clockwise from the outboard midplane, so a positive n_I^s corresponds to an upward shift of the impurity density and a negative n_I^s corresponds to a downward shift. Thus, we expect an upward shift to be produced by counter-injection and a downward shift to be produced by co-injection, and we expect a negative main ion pressure gradient to produce an upward shift which persists even in the absence of NBI.

The cos-component n_I^c is positive and is a symmetric function of beam power (independent of beam direction) in the absence of pressure gradients. A negative main ion pressure gradient produces a positive contribution ($\Delta n_I^c > 0$) for co-injection, which adds to the contribution due to NBI, and produces a negative contribution ($\Delta n_I^c < 0$) for counter-injection, which subtracts from the contribution due to NBI. A positive n_I^c corresponds to an outward shift of the impurity density and a negative n_I^c corresponds to an inward shift. Thus, in a tokamak plasma with the usual negative main ion pressure gradient, we would expect to see an outward shift of the impurities with strong co-injection, an inward shift with weak co- or counter-injection, and a reduction of the inward shift and finally an outward shift with increasing strength of counter-injection.

The above expressions for n_I^s and n_I^c differ from the corresponding results in the collisional regime via the expression used to evaluate v_{Ip}^0, hence Y. Equation (28) is quite different from the corresponding result [4] in the collisional regime.

The variations in n_i and n_e over the flux surface are of order α/z and can be neglected when $\alpha/z \ll 1$.

3.7. Transport fluxes

The parallel and normal (radial) components of Eq.(20) can be combined to obtain an expression for particle transport across flux surfaces

$$\langle \nabla\psi \cdot n_j \vec{v}_j \rangle = - \frac{\langle R^2 \nabla\phi \cdot (\vec{R}_j + \vec{N}_j)\rangle}{e_j}$$

Our previous results for the rotation velocities may be used to evaluate this expression for the large-aspect-ratio, low-beta equilibrium. It is important to retain $O(\epsilon^2)$ terms that were neglected in writing Eqs (26) and (28). (These terms are given in Ref. [4].) We write the radial impurity flux as a sum of contributions arising from different effects

$$\langle n_I v_{Ir}\rangle = \langle n_I v_{Ir}\rangle_{PS} + \langle n_I v_{Ir}\rangle_{NC} + \langle n_I v_{Ir}\rangle_M + \langle n_I v_{Ir}\rangle_I + \langle n_I v_{Ir}\rangle_{\Phi'} + \langle n_I v_{Ir}\rangle_{\tilde{\Phi}} \qquad (37)$$

The first two components correspond to the Pfirsch-Schlüter and neoclassical fluxes of the usual transport theory, but now modified to account for the radial transfer of momentum and for the variation of the impurity and ion densities over the flux surface.

$$\langle n_I v_{Ir}\rangle_{PS} = \frac{n_I^0 m_I \nu_{Ii}^0 \epsilon^2}{e_I B_p^0} \times \left[\left|\left(\frac{1+2q^2}{q^2}\right) + \left(\frac{n_i^c}{\epsilon}\right)\right| P_i' - \left|\left(\frac{1+2q^2}{q^2}\right) + \left(\frac{n_I^c}{\epsilon}\right)\right| (1+\beta_I) P_I'\right] \qquad (38)$$

$$\langle n_I v_{Ir}\rangle_{NC} = \frac{n_I^0 m_I \nu_{Ii}^0}{e_I B_p^0 d} \times \left[\left|(\hat{\mu}_i \hat{\mu}_I) + \epsilon^2 \hat{\mu}_i \left((1+\beta_I)\left(\frac{n_I^c}{\epsilon}\right) - \left(\frac{n_i^c}{\epsilon}\right)\right)\right| P_i' - \left|\hat{\mu}_I(\xi_i + \beta_I(1+\xi_i)) + \epsilon^2 \hat{\mu}_I \left(\left(\frac{n_i^c}{\epsilon}\right) - \left(\frac{n_I^c}{\epsilon}\right)\right)\right| P_I'\right] \qquad (39)$$

where

$$d \equiv \xi_i + \xi_I (1+\xi_i) \qquad (40)$$

For a negative main-ion density gradient ($P_i' < 0$), both of these flux components will be inward.

The third term in Eq.(37) is the transport flux resulting directly from the interaction of the beam ions with the main ions and impurities.

$$\langle n_I v_{Ir}\rangle_M = \frac{-1}{e_I B_p^0 d} \times \left[\hat{M}^0_{\phi I}\left\{\hat{\mu}_I(1+\xi_i) + \epsilon^2\left(d+(1+d)\left(\frac{n_I^c}{\epsilon}\right) -(1+\xi_i)\left(\frac{n_i^c}{\epsilon}\right)\right)\right\} + \hat{M}^0_{\phi i}\left\{\hat{\mu}_I + \epsilon^2\left(\left(1+\tfrac{1}{2}(\beta_I+\xi_I)\right)\left(\frac{n_I^c}{\epsilon}\right) - \left(\frac{n_i^c}{\epsilon}\right)\right)\right\}\right] \quad (41)$$

This contribution to the impurity flux is inward for co-injection and outward for counter-injection.

The fourth term in Eq.(37) results from retention of the inertial term ($nm(\vec{v}\cdot\nabla)\vec{v}$) in the momentum balance equations, which produces $O(\epsilon^2)$ contributions to the expressions for the toroidal and poloidal rotation velocities, which in turn contribute $O(\epsilon^2)$ terms in the transport flux.

$$\langle n_I v_{Ir}\rangle_I \equiv -\frac{n_I^0 m_I \nu_{Ii}^0 \epsilon^2 B_\phi^0}{e_I B_p^0 (B^0)^2 d}\left[(\xi_i+\beta_I(1+\xi_i))\langle \hat{G}_I\rangle - \hat{\mu}_I\langle \hat{G}_i\rangle\right] \quad (42)$$

where

$$\langle \hat{G}_I\rangle \equiv \frac{1}{2}\left(\frac{B_\phi^0}{qR_0\nu_{Ii}^0}\right) \times \left[\left\{\left(\frac{B_\phi^0}{B_p^0}v_{Ip}^0\right)^2 + \omega^2 + 2\delta_1\omega\right\}\left(\frac{n_I^s}{\epsilon}\right) + \left\{2\delta_2\omega\right\}\left(\frac{n_i^s}{\epsilon}\right)\right] \quad (43)$$

The quantities δ_1 and δ_2 are defined as follows:

$$\delta_1 \equiv \frac{(\alpha/z)}{(1+(\alpha/z))e_i B_p^0} \times \left[\frac{1}{n_I^0}\frac{\partial p_I^0}{\partial r} - \frac{T}{(1+(\alpha/z))}\left\{\left(\frac{\alpha}{z}\right)\frac{1}{n_I^0}\frac{\partial n_I^0}{\partial r} + \frac{1}{n_i^0}\frac{\partial n_i^0}{\partial r}\right\}\right]$$

$$\delta_2 \equiv \frac{1}{(1+(\alpha/z))e_i B_p^0} \times \left[\frac{1}{n_i^0}\frac{\partial p_i^0}{\partial r} - \frac{T}{(1+(\alpha/z))}\left\{\frac{1}{n_I^0}\frac{\partial n_I^0}{\partial r} + \frac{1}{n_i^0}\frac{\partial n_i^0}{\partial r}\right\}\right]$$

The expression for $\langle \hat{G}_i\rangle$ is obtained from Eq.(43) by exchanging the i and I subscripts, including the expression in the definition of ω, and exchanging δ_1 and δ_2. The quantity δ_1 is of order α/z and may be neglected in most cases. Note that $\langle \hat{G}_I\rangle$ depends (directly and indirectly through n_I^s) on the radial electric field, so that the inertial flux is actually due in large part to the radial electric field. This inertial effect will produce an outward impurity flux when the impurity density is shifted down ($n_I^s < 0$), which occurs for strong co-injection. Conversely, strong counter-injection will produce an inward flux contribution via this inertial effect.

The fifth term in Eq.(37) is the radial impurity flux driven by the linear component of the radial electric field.

$$\langle n_I v_{Ir}\rangle_{\Phi'} = \frac{n_I^0 m_I \nu_{Ii}^0}{e_I B_p^0}\left[\hat{\mu}_I\gamma_I + \epsilon^2\left\{\beta_I\left(\frac{1+2q^2}{q^2}\right) + (\beta_I+\gamma_I)\left(\frac{n_I^c}{\epsilon}\right) - \gamma_I\left(\frac{n_i^c}{\epsilon}\right)\right\}\right]\left(\frac{E_r^0}{B_p^0}\right) \quad (44)$$

where

$$\gamma_I \equiv \frac{\beta_i + \beta_I(1+\xi_i)}{\xi_i + \xi_I(1+\xi_i)}$$

This term will have the same sign as the radial electric field. Thus, it will produce an outward contribution to the impurity flux for strong co-injection and an inward contribution for counter-injection and weak co-injection (assuming $P_i' < 0$).

The last term in Eq.(37) is the impurity transport flux driven by the poloidal variation of the potential over the flux surface, or the poloidal electric field.

$$\langle n_I v_{Ir}\rangle_{\tilde{\Phi}} = -\frac{n_I^0 m_I \nu_{Ii}^0 \epsilon^2}{e_I B_p^0}\left[\beta_I\left\{\delta_1\left(\frac{n_I^c}{\epsilon}\right) + \delta_2\left(\frac{n_i^c}{\epsilon}\right)\right\} - \frac{1}{2}\frac{n_I^0 B_\phi^0 zT(\xi_I + \xi_i(1+\beta_I))}{e_I (B^0)^2(1+\alpha/z)d}\left\{\left(\frac{n_I^s}{\epsilon}\right)\left(\frac{n_i^c}{\epsilon}\right) - \left(\frac{n_i^s}{\epsilon}\right)\left(\frac{n_I^c}{\epsilon}\right)\right\}\right] \quad (45)$$

This term is of order α/z and usually may be neglected.

The main ion transport flux may be obtained by exchanging the i and I subscripts in the above equations. In addition, in Eq.(45), δ_1 and δ_2 must be exchanged, $\alpha/z \to z/\alpha$ and $z \to 1$.

Note that Eqs (38) and (39) describe diffusive transport fluxes, i.e. fluxes proportional to a pressure gradient, while Eqs (41), (42), (44) and (45) describe convective transport fluxes arising from viscous and inertial forces and the direct momentum input. The diffusive impurity fluxes are inward for the normal negative main ion density gradient. The rotational and electric field contributions to the convective flux are outward for strong co-injection and inward for counter-injection. The direct momentum input contribution of Eq.(41) to the convective flux is inward for strong co-injection and outward for counter-injection. Thus, with strong co-injection, the outward impurity fluxes produced by the rotation and radial electric field compete with the inward impurity flux produced by the main ion pressure gradient. With counter-injection, all three components are inward. The other flux components tend to be less significant (see Section 4).

We note that the inertial flux of Eq.(42) and the flux of Eq.(45) are $O(\epsilon^2)$ and thus are only significant when at least the impurities are collisional. We further note that the transport fluxes of Eqs (38) and (39) retain the principle of detailed balance [14, 15], stating that when the impurities are collisional, the main ion flux will also be collisional (i.e. $O(\epsilon^2)$). However, the flux component driven by E_r does not retain this principle. The main ion flux, obtained by interchanging the i and I subscripts, scales with the plateau regime viscosity (i.e. $O(\hat{\mu}_i)$).

These transport fluxes differ considerably from the results [4] obtained for the collisional regime, to which they reduce in the limit $\hat{\mu}_i$, $\hat{\mu}_I \to 0$. The neoclassical flux of Eq.(39) vanishes and the other fluxes are modified substantially in the collisional limit.

4. MODEL PROBLEMS

In order to illustrate the nature of the theoretical results developed in the previous two sections, we now apply the formalism to two model problems with deuterium plasmas representative of the interior regions of the ISX-B and PLT tokamaks. Neutral-beam injection at 40 keV was considered in both cases. In both of these machines the toroidal rotation velocity with directed NBI has been measured [7, 10], so that we can partially extract the anomalous viscous drag frequencies ν_{di} and ν_{dI} from the experimental data. The procedure we follow is first to calculate the ratio β_I/β_i from Eq.(33) and then to determine β_i from the summed toroidal momentum balance, which may be rearranged to obtain

$$\beta_i \approx \frac{M_\phi}{n_i m_i \nu_{iI} v_\phi (1 + \beta_I/\beta_i)} \tag{46}$$

where M_ϕ is the total toroidal momentum input of the beam and v_ϕ is the measured rotation velocity. Usually, only one rotation velocity, that of an impurity species, is measured. For the purpose of determining β_i, we take this impurity rotation velocity approximately as a common rotation velocity for all species. If the rotation velocities of both the ions and impurities are measured, then Eqs (30) and (31) can be used to determine ν_{di} and ν_{dI} separately.

We examined a deuterium plasma in ISX-B, with a titanium impurity with $\alpha = 0.05$. We chose parameters typical of an ISX plasma at $r \cong 10$ cm ($n_e = 2.8 \times 10^{13}$ cm^{-3}, $T_e = T_i = 430$ eV, $q = 1.2$, $((1/p)(\partial p/\partial r))^{-1} = -8.5$ cm) and computed the toroidal momentum input of the H^0 beam with a beam deposition and Fokker-Planck slowing-down code. For this ISX plasma, the beam momentum input was mostly to the deuterium, $|\hat{M}_i| \gg |\hat{M}_I|$. The deuterium was in the banana plateau regime, with viscosity $\hat{\mu}_i \approx O(\sqrt{\epsilon})$, and the titanium was in the collisional regime, with $\hat{\mu}_I \lesssim O(\epsilon^2)$. The anomalous drag coefficients computed in the manner described above led to the ratio $\beta_I/\beta_i = 0.36$, using the toroidal rotation velocities measured at 1 MW co-injection.

The rotation velocities and the radial electric field computed for the ISX model over a range of co- and counter-injected beam powers are shown in Fig.1. The toroidal rotation velocities and their difference scale linearly with beam power, in accordance with Eqs (26′) and (27). (In the model problems we have not accounted for temperature differences associated with different levels of beam power.) The poloidal rotation velocities vary with beam power in a more interesting manner. Since circulating ions tend to move along the field lines, this leads to a positive poloidal rotation with co-injection and a negative poloidal rotation with counter-injection. However, the $E_r \times B_\phi$ drift of the ions leads to a negative poloidal rotation when $E_r > 0$ and to a positive poloidal rotation when $E_r < 0$. In general, E_r is negative for counter-injection and becomes positive only when the co-injected momentum contribution to Eq.(25) is large enough to offset the negative pressure gradient contribution. The poloidal velocities must satisfy the lowest-order parallel momentum balance

$$\mu_I v_{Ip}^0 + \mu_i v_{ip}^0 \approx O(\epsilon^2)$$

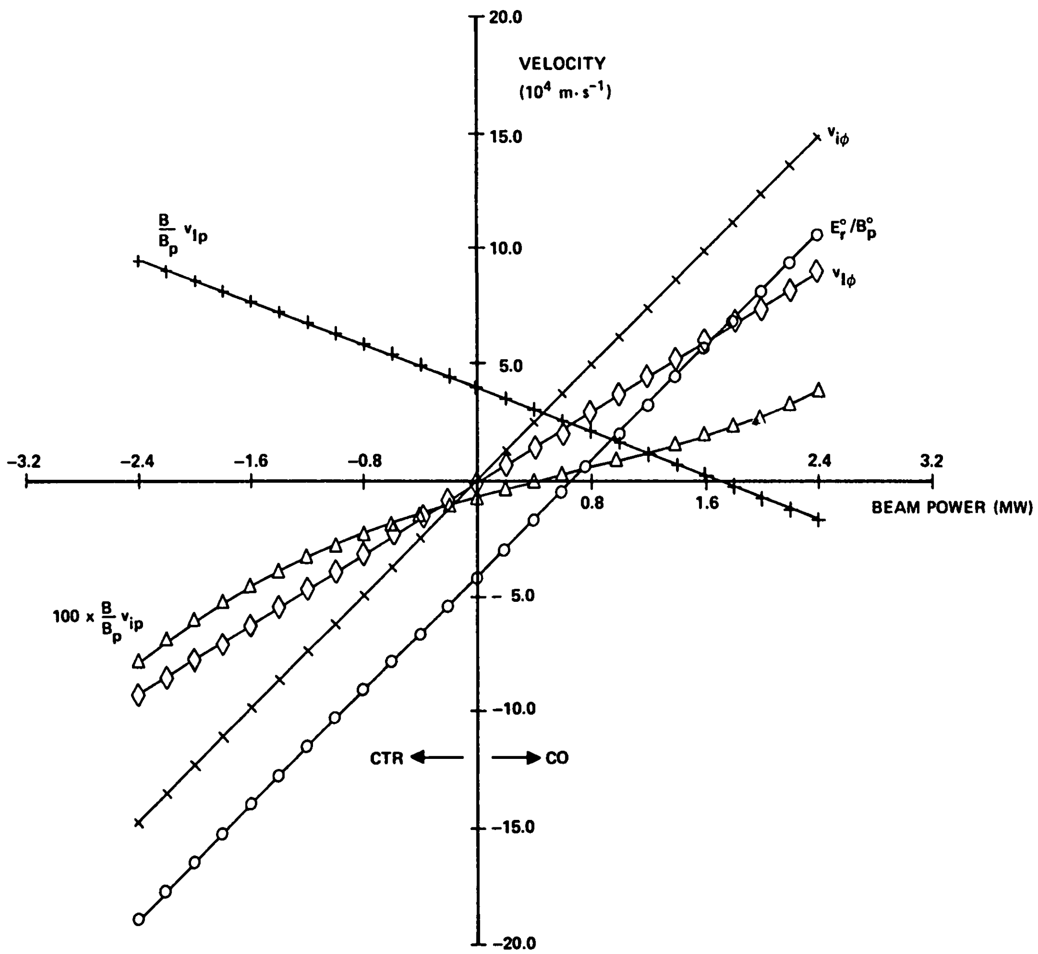

FIG.1. Rotation velocities and radial electric field for the ISX-B model problem.

Thus, v^0_{Ip} and v^0_{ip} must be oppositely directed and, since $M_I \ll M_i$, $|v^0_{ip}| \ll |v^0_{Ip}|$. The effect of the strong-rotation correction ($\approx v_\phi/v_{th}$) on $\hat{\mu}_i$ is apparent from the curvature in v^0_{ip}. This mixed-regime result is very different from the collisional regime result found previously (see Ref. [4], Fig.5) for the poloidal velocities.

The amplitudes of the sine and cosine components of the poloidal variation of the impurity density are shown in Fig.2. The general dependence upon beam power is consistent with the general discussion of the previous section. We note that there is a strong inward and a slight upward shift with strong counter-injection, which changes to a strong downward and a slight outward shift with strong co-injection. We further note that a rather dramatic change in the poloidal distribution of impurities takes place over a small range of co-injected beam powers about the power for which the impurity poloidal velocity changes sign.

The components of the radial impurity transport flux are shown in Fig.3. Most of these flux components are sensitive functions of the radial electric field and of the poloidal distribution of the impurities, which in turn is also strongly dependent upon E_r. Since $\alpha/z \ll 1$, the poloidal variation of the main ion density and of the electric potential can be neglected. The neoclassical component of Eq.(39) varies with beam power primarily as the (n^c_I/ϵ) term multiplying P'_i; since $P'_i < 0$, this neoclassical component has the opposite sign of (n^c_I/ϵ). The Pfirsch-Schlüter component of Eq.(38) is inward and relatively insensitive to the beam power. The sum of these two terms is shown as $\langle n_I v_{Ir} \rangle_\nabla$. Since $|M_{\phi i}| \gg |M_{\phi I}|$ and $\hat{\mu}_I$ is small, the $(n^c_I/\epsilon)\hat{M}^0_{\phi i}$ term is the dominant term. The inertial component $\sim -\langle G_I \rangle \sim -(n^s_I/\epsilon)$ and is a strong function of the radial electric field. This component dominates the others for strong co- or counter-injection. The impurity flux driven by the linear component of the radial electric field of Eq.(44) is reduced in magnitude

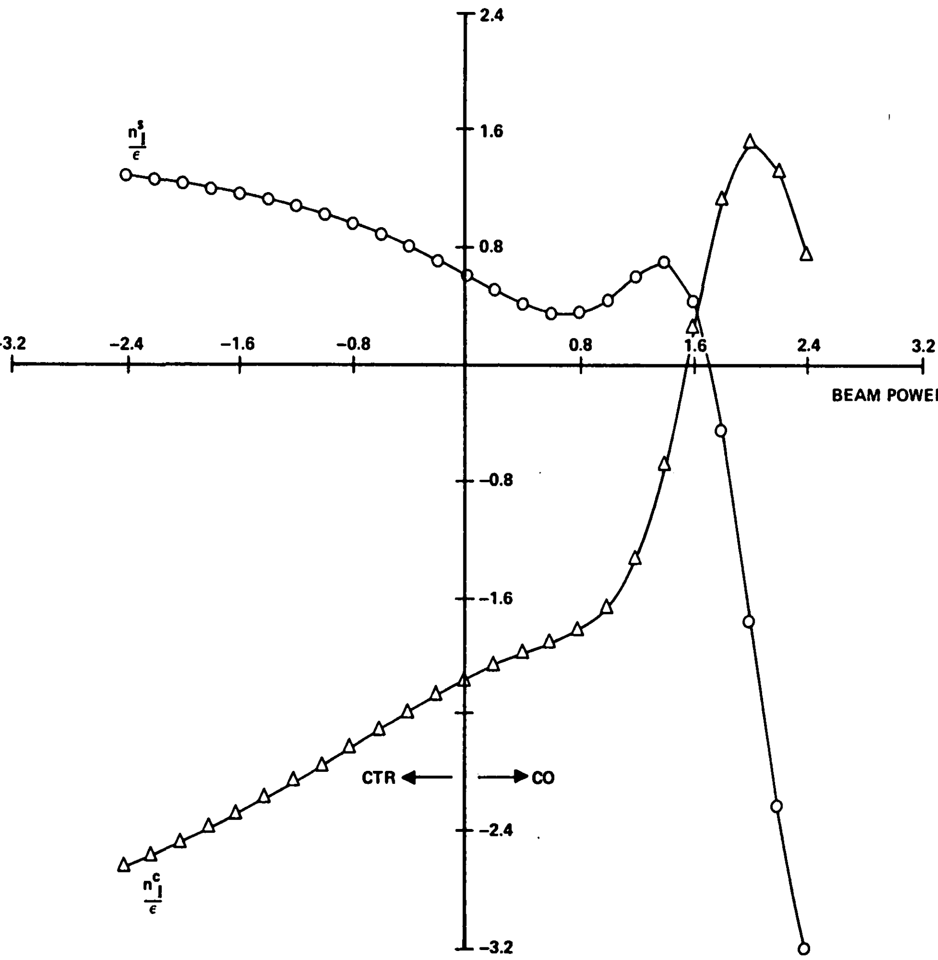

FIG.2. Poloidal variation of impurity density for the ISX-B model problem.

by the negative (n_I^s/ϵ) term over much of the range of beam injection considered. This result for the total transport flux is similar to the result found in the collisional regime [4], but now the dominant component is $\langle n_I v_{Ir} \rangle_I$ rather than $\langle n_I v_{Ir} \rangle_{\Phi'}$.

The different flux components tend to add up for counter-injection and to cancel for co-injection. Moreover, the inertial flux component is large and inward even for small values of counter-injected beam power, but becomes large and outward only for large values of co-injected beam power. The total impurity flux is inward without neutral-beam injection, primarily because of the pressure-gradient-driven Pfirsch-Schlüter and neoclassical fluxes but also because of inward contributions from the inertial and linear radial electric field components. For counter-injection, the magnitude of the inward impurity flux rapidly (stronger than linear) increases with increasing beam power. On the other hand, with co-injection, the inward impurity flux becomes only slowly reduced in magnitude as the beam power increases, and finally it changes to an outward flux which is a strongly increasing function of beam power after the co-injected beam power is increased beyond a certain value, about 1.8 MW in this ISX model problem.

The functional dependence of the radial titanium flux shown in Fig.3 is in reasonable agreement with experimental observations [8] of the accumulation of titanium in the centre of the ISX-B plasma. During Ohmic discharges, the radiated power increases throughout the shot, indicating an increasing central titanium concentration and an inward titanium flux. With 1 MW of counter-injected beam, the radiated power increases sharply, and a disruption occurs shortly thereafter, indicating a sharply increased central titanium accumulation and a much larger inward titanium flux relative to the Ohmic discharge. On the other hand, with 1 MW of co-injected beam, the radiated power remains almost constant during the discharge, indicating a constant central titanium con-

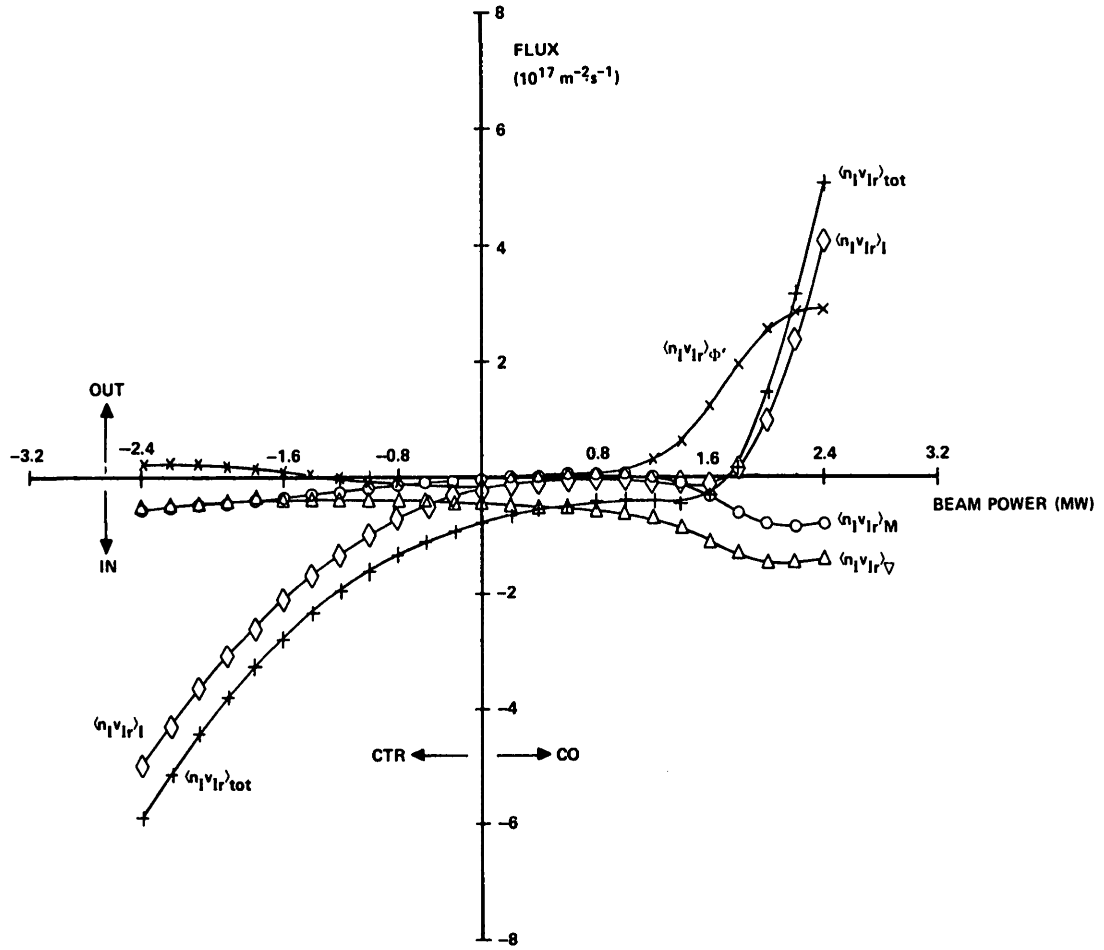

FIG.3. Impurity transport fluxes for the ISX-B model problem.

centration and a very small titanium flux. When the co-injected beam power is increased to 1.2 MW, the radiated power actually decreases, indicating a decreasing central titanium concentration and an outward titanium flux.

As a second model problem, we chose a deuterium plasma with parameters representative of PLT at $r = 10{-}20$ cm ($n_i = 3 \times 10^{13}$ cm^{-3}, $T_i = T_e = 1$ keV, $q = 1.5$, $((1/p_i)(\partial p_i/\partial r))^{-1} = -30$ cm). We used the previously described procedure and the experimentally determined [10] rotation velocities to determine the values $\nu_{di} = 34$ s^{-1}, $\nu_{dI} = 5100$ s^{-1} and $\beta_I/\beta_i = 1.5$. We considered a uniform tungsten impurity of concentration $\alpha = 0.1$ and injection of a 40 keV D^0 beam, with the momentum input calculated as described for the first model problem. The deuterium ions are in the banana-plateau regime and the tungsten ions are in the collisional regime ($\mu_I \to 0$).

Our results for the impurity transport fluxes in the PLT model problem are shown in Fig.4. The qualitative results are similar to those for ISX, but the quantitative results are quite different for the two cases. We note that the toroidal rotation velocity is larger in PLT than in ISX, for a given directed beam power, but the total number of plasma particles is greater in PLT, implying a stronger radial momentum transfer mechanism operating in ISX than in the much larger PLT.

We note that in the PLT model problem with no injection the inward impurity flux due to the radial electric field (the linear contribution of Eq.(44) plus the non-linear, inertial contribution of Eq.(42)) is of the same size as the inward contribution due to the pressure gradient of Eqs (38) and (39). Thus, the total inward impurity flux is about twice the standard neoclassical value, and the outward deuterium flux is about twice the neoclassical value, which is reminiscent of the observation that the ion heat conductivity inferred from experiment is about 2–4 times the neoclassical value. Possibly, the radial electric field

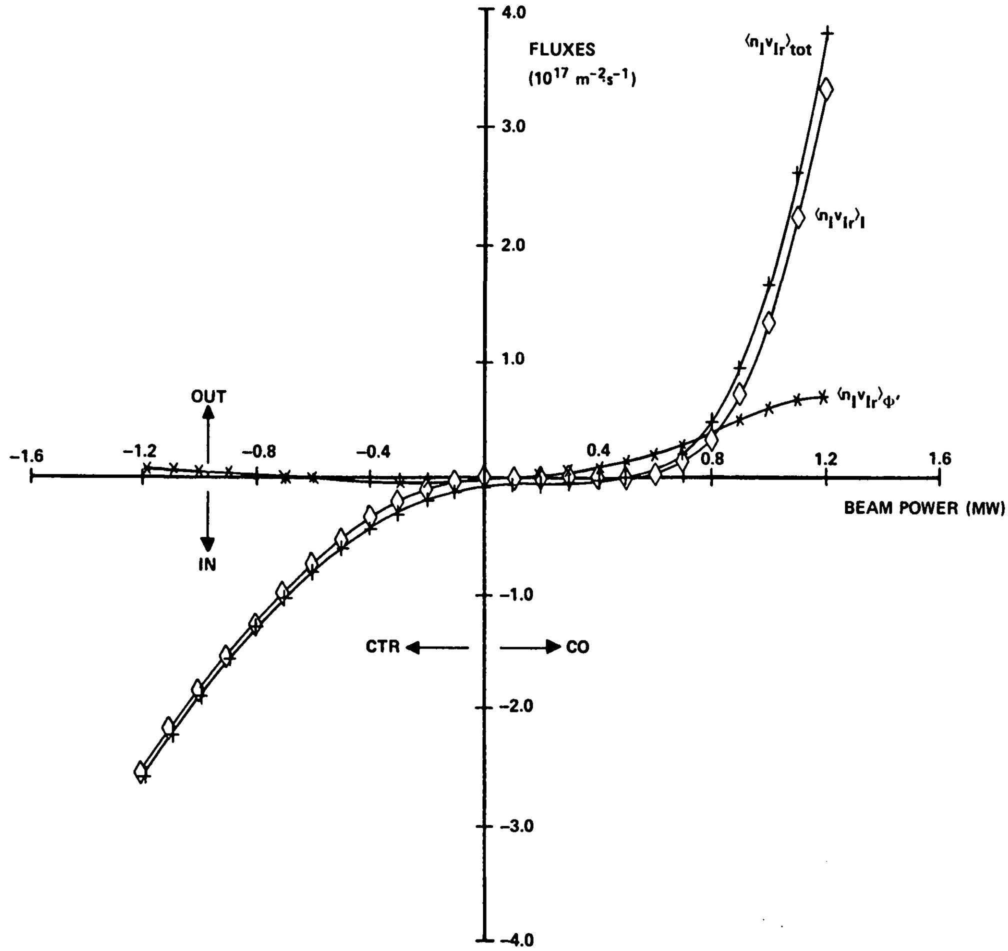

FIG.4. Impurity transport fluxes for the PLT model problem.

effects of the present theory also produce a convective heat flux which, if taken into account, would in part explain this well-known 'factor of 2–4' disparity between neoclassical predictions and experiment.

The total tungsten fluxes in Fig.4 are in reasonable agreement with those measured in PLT [5]. The experimental fluxes at $10 \leqslant r \leqslant 20$ cm were about $(6-7) \times 10^{15}\ m^{-2} \cdot s^{-1}$ inward without NBI, about $(2-4) \times 10^{15}\ m^{-2} \cdot s^{-1}$ outward with 585 kW co-injection, and about $(25-50) \times 10^{15}\ m^{-2} \cdot s^{-1}$ inward with 430 kW counter-injection. The total calculated tungsten fluxes shown in Fig.4 are about $9 \times 10^{15}\ m^{-2} \cdot s^{-1}$ inward without NBI, about $4 \times 10^{15}\ m^{-2} \cdot s^{-1}$ outward with 585 kW co-injection, and about $50 \times 10^{15}\ m^{-2} \cdot s^{-1}$ inward with 430 kW counter-injection.

While the reasonable agreement between our model problem calculations and the experimental observations in ISX-B and PLT is encouraging, we emphasize that these are model problem calculations, not analyses of these specific experiments. We have not attempted to take into account such important effects as differences in plasma temperature for different injected beam powers, changes in temperature profiles during a discharge and their effect upon the parameters, etc., and we do not include temperature gradient effects in our model. Such an analysis is in progress.

The implications of these model problem calculations for TFTR and subsequent tokamaks with directed NBI are now apparent. Co-injected NBI is predicted to produce an outward component of the impurity flux which, for sufficiently large NBI power, is capable of overcoming the normal inward flux driven by the pressure gradient. Since NBI momentum deposition would be large in the central region where the pressure gradient is small, and NBI momentum input would be small in the outer region where the pressure gradient is large, it could be anticipated that co-injection inhibits the penetration of edge-originated impurities to the plasma centre but cannot prevent their entering the outer regions of the plasma. Thus, one would expect that the impurities are confined largely to the edge region, thereby creating a cool, radiating edge which would reduce the sputtering erosion of the limiter and first wall.

Moreover, since we have linked the impurity transport to the radial electric field and shown the effect to be substantial, our results suggest the possibility of controlling impurities by manipulating the radial electric field by means other than NBI.

ACKNOWLEDGEMENTS

The work of W.M. Stacey and A.W. Bailey was supported by the Callaway Foundation and by the United States Department of Energy, under contract with the Georgia Institute of Technology, and the work of D.J. Sigmar and K.C. Shaing was supported by the United States Department of Energy, under contract with the Union Carbide Corporation.

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(Manuscript received 9 April 1984
Final manuscript received 15 February 1985)